Math 75 Quiz 3 - v. 1 Solutions

1. **Multiple Choice.** (5 points) *Circle the letter of the best answer.*

   $\int_{-2}^{1} \frac{1}{x^4}\, dx =$

   (a) $-\frac{3}{8}$

   (b) $-\frac{33}{8}$

   (c) $\boxed{\text{does not exist}}$

   (d) none of these.

   $\frac{1}{x^4}$ is not defined at $x = 0$, which is in the interval $[-2, 1]$. Therefore the integral is not defined.

2. **Multiple Choice.** (5 points) *Circle the letter of the best answer.*

   If $F(x) = \int_{-\pi}^{x^2} \sin(t^4)\, dt$, then $F'(x) =$

   (a) $\sin(x^4)$

   (b) $\boxed{2x \sin(x^8)}$

   (c) $2x \cos(x^8)$

   (d) $-2x \cos(x^8)$

   Using the Fundamental Theorem of Calculus (part 1) and the chain rule, we have

$$\begin{aligned} F'(x) &= \sin((x^2)^4) \cdot 2x \\ &= 2x \sin(x^8). \end{aligned}$$

3. **Multiple Choice.** (5 points) *Circle the letter of the best answer.*

If $\int_3^5 f(x)\,dx = -1$ and $\int_4^5 f(x)\,dx = -2$, then $\int_3^4 f(x)\,dx =$

(a) $\boxed{1}$

(b) -1

(c) 2

(d) -3

Notice that if we interpret the definite integral as an area, we see that

$$\int_3^4 f(x)\,dx = \int_3^5 f(x)\,dx - \int_4^5 f(x)\,dx.$$

Therefore

$$\int_3^4 f(x)\,dx = -1 - (-2) = 1.$$

4. (10 points) Evaluate $\int x\cos(5x^2)\,dx$.

Let $u = 5x^2$. Then $du = 10x\,dx$. Therefore

$$\begin{aligned}\int x\cos(5x^2)\,dx &= \frac{1}{10}\int 10x\cos(5x^2)\,dx\\ &= \frac{1}{10}\int \cos u\,du\\ &= \frac{1}{10}\sin u + C\\ &= \frac{1}{10}\sin(5x^2) + C.\end{aligned}$$

5. (10 points) Evaluate $\int_1^2 \sqrt{3t-2}\,dt$.

The domain of $\sqrt{3t-2}$ is $t \geq \frac{2}{3}$ and $\sqrt{3t-2}$ is continuous on its domain. Therefore the integral is defined, and the Fundamental Theorem of Calculus (part 2) applies.

Let $u = 3t - 2$. Then $du = 3\,dt$.

"Ignore-the-Problem" Method.

$$\begin{aligned}
\int_1^2 \sqrt{3t-2}\,dt &= \frac{1}{3}\int_1^2 3\sqrt{3t-2}\,dt \\
&= \frac{1}{3}\int_?^? \sqrt{u}\,du \\
&= \frac{1}{3}\cdot\frac{2}{3}u^{3/2}\bigg|_?^? \\
&= \frac{2}{9}(3t-2)^{3/2}\bigg|_1^2 \\
&= \frac{2}{9}\left((3(2)-2)^{3/2} - (3(1)-2)^{3/2}\right) \\
&= \frac{2}{9}\left(4^{3/2} - 1^{3/2}\right) \\
&= \frac{2}{9}(8-1) \\
&= \frac{14}{9}.
\end{aligned}$$

"Change-the-Limits" Method.

When $t = 1$, $u = 3(1) - 2 = 1$.

When $t = 2$, $u = 3(2) - 2 = 4$.

Therefore

$$\begin{aligned}
\int_1^2 \sqrt{3t-2}\,dt &= \frac{1}{3}\int_1^2 3\sqrt{3t-2}\,dt \\
&= \frac{1}{3}\int_1^4 \sqrt{u}\,du \\
&= \frac{1}{3}\cdot\frac{2}{3}u^{3/2}\bigg|_1^4 \\
&= \frac{2}{9}\left(4^{3/2} - 1^{3/2}\right) \\
&= \frac{2}{9}(8-1) \\
&= \frac{14}{9}.
\end{aligned}$$

6. **BONUS.** (2 points) Evaluate $\displaystyle\int_{-1}^{1} \frac{x^5}{\cos x}\, dx$.

$f(x)$ is defined on the interval $[-1, 1]$ since $\cos x$ is not equal to 0 for any x in that interval.

Notice that $f(x) = \dfrac{x^5}{\cos x}$ is an odd function, since

$$\begin{aligned} f(-x) &= \frac{(-x)^5}{\cos(-x)} \\ &= \frac{(-x)^5}{\cos x} && \text{(Since } \cos x \text{ is an even function)} \\ &= -\frac{x^5}{\cos x} \\ &= -f(x). \end{aligned}$$

The regions under the graph of $f(x)$ between -1 and 1 have the same area above the x-axis as below. Therefore the net area is $\displaystyle\int_{-1}^{1} \frac{x^5}{\cos x}\, dx = 0$.