Math 75 Quiz 3 - v. 2 Solutions

1. **Multiple Choice.** (5 points) *Circle the letter of the best answer.*

   If $\int_3^5 f(x)\,dx = -1$ and $\int_4^5 f(x)\,dx = -2$, then $\int_3^4 f(x)\,dx =$

   (a) $\boxed{1}$
   (b) -1
   (c) 2
   (d) -3

   Notice that if we interpret the definite integral as an area, we see that

   $$\int_3^4 f(x)\,dx = \int_3^5 f(x)\,dx - \int_4^5 f(x)\,dx.$$

   Therefore

   $$\int_3^4 f(x)\,dx = -1 - (-2) = 1.$$

2. **Multiple Choice.** (5 points) *Circle the letter of the best answer.*

   $\int_{-2}^{1} \frac{1}{x^4}\,dx =$

   (a) $-\frac{3}{8}$
   (b) $-\frac{33}{8}$
   (c) $\boxed{\text{does not exist}}$
   (d) none of these.

   $\frac{1}{x^4}$ is not defined at $x = 0$, which is in the interval $[-2, 1]$. Therefore the integral is not defined.

3. **Multiple Choice.** (5 points) *Circle the letter of the best answer.*

If $F(x) = \int_{-\pi}^{x^2} \sin(t^4)\, dt$, then $F'(x) =$

(a) $\sin(x^4)$

(b) $\boxed{2x\sin(x^8)}$

(c) $2x\cos(x^8)$

(d) $-2x\cos(x^8)$

Using the Fundamental Theorem of Calculus (part 1) and the chain rule, we have

$$\begin{aligned} F'(x) &= \sin((x^2)^4) \cdot 2x \\ &= 2x\sin(x^8). \end{aligned}$$

4. (10 points) Evaluate $\int_1^2 \sqrt{3t-2}\, dt$.

The domain of $\sqrt{3t-2}$ is $t \geq \frac{2}{3}$ and $\sqrt{3t-2}$ is continuous on its domain. Therefore the integral is defined, and the Fundamental Theorem of Calculus (part 2) applies.

Let $u = 3t-2$. Then $du = 3\, dt$.

"Ignore-the-Problem" Method.

$$\begin{aligned} \int_1^2 \sqrt{3t-2}\, dt &= \frac{1}{3}\int_1^2 3\sqrt{3t-2}\, dt \\ &= \frac{1}{3}\int_?^? \sqrt{u}\, du \\ &= \frac{1}{3} \cdot \frac{2}{3} u^{3/2}\Big|_?^? \\ &= \frac{2}{9}(3t-2)^{3/2}\Big|_1^2 \\ &= \frac{2}{9}\left((3(2)-2)^{3/2} - (3(1)-2)^{3/2}\right) \\ &= \frac{2}{9}\left(4^{3/2} - 1^{3/2}\right) \\ &= \frac{2}{9}(8-1) = \frac{14}{9}. \end{aligned}$$

"Change-the-Limits" Method.

When $t = 1$, $u = 3(1) - 2 = 1$.

When $t = 2$, $u = 3(2) - 2 = 4$.

Therefore

$$\begin{aligned}\int_1^2 \sqrt{3t-2}\,dt &= \frac{1}{3}\int_1^2 3\sqrt{3t-2}\,dt \\ &= \frac{1}{3}\int_1^4 \sqrt{u}\,du \\ &= \frac{1}{3}\cdot\frac{2}{3}u^{3/2}\Big|_1^4 \\ &= \frac{2}{9}\left(4^{3/2} - 1^{3/2}\right) \\ &= \frac{2}{9}(8-1) = \frac{14}{9}.\end{aligned}$$

5. (10 points) Evaluate $\displaystyle\int x\cos(5x^2)\,dx$.

   Let $u = 5x^2$. Then $du = 10x\,dx$. Therefore

$$\begin{aligned}\int x\cos(5x^2)\,dx &= \frac{1}{10}\int 10x\cos(5x^2)\,dx \\ &= \frac{1}{10}\int \cos u\,du \\ &= \frac{1}{10}\sin u + C \\ &= \frac{1}{10}\sin(5x^2) + C.\end{aligned}$$

6. **BONUS.** (2 points) Evaluate $\displaystyle\int_{-1}^{1} \frac{x^5}{\cos x}\,dx$.

   $f(x)$ is defined on the interval $[-1, 1]$ since $\cos x$ is not equal to 0 for any x in that interval.

   Notice that $f(x) = \dfrac{x^5}{\cos x}$ is an odd function, since

$$\begin{aligned}f(-x) &= \frac{(-x)^5}{\cos(-x)} \\ &= \frac{(-x)^5}{\cos x} && \text{(Since } \cos x \text{ is an even function)} \\ &= -\frac{x^5}{\cos x} = -f(x).\end{aligned}$$

   The regions under the graph of $f(x)$ between -1 and 1 have the same area above the x-axis as below. Therefore the net area is $\displaystyle\int_{-1}^{1} \frac{x^5}{\cos x}\,dx = 0$.