Section 5.1 - Areas and Distances - lecture 1, p. 313 Stewart, 4th Ed.

The distance problem.

Question. If we know the velocity of an object for a certain length of time, can we figure out how far it goes in that time?

Sure, you say: rate $\times$ time = distance. But wait! This only works if the velocity (rate) is constant. What to do if it isn't?

We can **estimate** the distance with "rectangles."

Example 4, p. 374. Suppose the odometer on our car is broken and we want to estimate the distance driven over a 30-second time interval. We take speedometer readings every five seconds and record them in the following table, along with conversions to ft/sec to unify things:

Time (sec)	0	5	10	15	20	25	30
Velocity (mi/hr)	17	21	24	29	32	31	28
Velocity (ft/sec)	25	31	35	43	47	46	41

The trick to the estimation is to treat the velocity as constant over each 5-second interval.

Left endpoints: between $t = 0$ and $t = 5$, assume $v = 25$. Then distance traveled in the first 5 seconds is approximately ______________________ ft. Computing similarly the rest of the distances, we get

$$\begin{aligned}\text{total distance} &\approx \_\_\_\_ \cdot \_\_\_\_ + \_\_\_\_ \cdot \_\_\_\_ + \_\_\_\_ \cdot \_\_\_\_ + \_\_\_\_ \cdot \_\_\_\_ \\ &\qquad + \_\_\_\_ \cdot \_\_\_\_ + \_\_\_\_ \cdot \_\_\_\_ \\ &= (\_\_\_\_ + \_\_\_\_ + \_\_\_\_ + \_\_\_\_ + \_\_\_\_ + \_\_\_\_) \cdot \_\_\_\_ \\ &= \_\_\_\_\_\_\_\_ \text{ ft.}\end{aligned}$$

Right endpoints: between $t = 0$ and $t = 5$, assume $v = 31$. Then distance traveled in the first 5 seconds is approximately ______________________ ft. Computing similarly the rest of the distances, we get

$$\begin{aligned}\text{total distance} &\approx \_\_\_\_ \cdot \_\_\_\_ + \_\_\_\_ \cdot \_\_\_\_ + \_\_\_\_ \cdot \_\_\_\_ + \_\_\_\_ \cdot \_\_\_\_ \\ &\qquad + \_\_\_\_ \cdot \_\_\_\_ + \_\_\_\_ \cdot \_\_\_\_ \\ &= (\_\_\_\_ + \_\_\_\_ + \_\_\_\_ + \_\_\_\_ + \_\_\_\_ + \_\_\_\_) \cdot \_\_\_\_ \\ &= \_\_\_\_\_\_\_\_ \text{ ft.}\end{aligned}$$

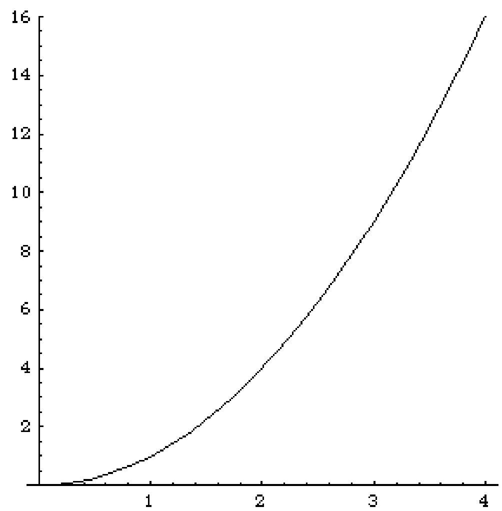
The actual distance is probably somewhere between ____________ and ____________ ft. (unless we drove erratically!).

Pictures of these estimates:

In general, we can estimate the area under a curve using rectangles. And from the velocity/distance problem, we just might suspect that it has something to do with **antiderivatives**!

Example. Find the area under $f(x) = x^2$ from $x = 1$ to $x = 4$.

Idea: estimate using rectangles. Divide the area under x^2 from 1 to 4 into 3 "rectangles."



Left endpoints (underestimate): area = ____.

Right endpoints (overestimate): area = ____.

Midpoints (closer): area = ____.

Actual area: 21 (we will learn to compute this later).

Fact. Our knowledge of antiderivatives will enable us to calculate the *exact* area using a *limiting process*.