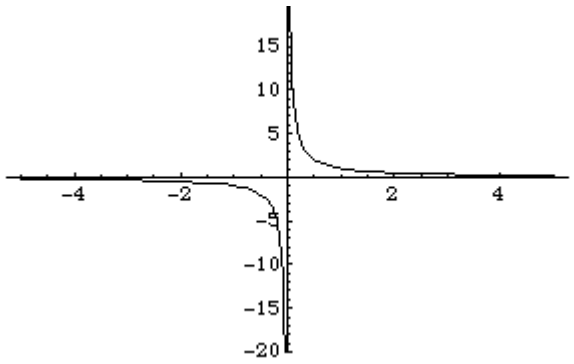
Math 75B Worksheet — Exploring Horizontal Asymptotes

Recall that the graph of $f(x) = \frac{1}{x}$ looks like $\longrightarrow$



1. Draw graphs of each of the following functions:
   (a) $g(x) = \frac{1}{x} + 3$
   (b) $h(x) = \frac{3}{x-2}$
   (c) $k(x) = -\frac{3}{x+1} + 4$

2. Turn to section 5-A.1 (Ebersole) for a review of horizontal asymptotes, if you need to.
   (a) Does $f(x)$ have a horizontal asymptote? If so, where? (Your answer should look like "$y =$ _____".)
   (b) Answer the above question for $g(x)$, $h(x)$, and $k(x)$ as in #1.

3. Recall that $\lim_{x \to a} f(x) = L$ means that as x gets close to a, the y-value of $f(x)$ tries to get to L. Looking at the graphs of $f(x)$, $g(x)$, $h(x)$, and $k(x)$,
   (a) What do you think should be the answer to $\lim_{x \to \infty} \left(\frac{1}{x} \right)$?
   (b) Find $\lim_{x \to \infty} \left(\frac{1}{x} + 3 \right)$, $\lim_{x \to \infty} \left(\frac{3}{x-2} \right)$, and $\lim_{x \to \infty} \left(-\frac{3}{x+1} + 4 \right)$.
   (c) Do the answers change if ∞ is replaced by $-\infty$?
   (d) What is the connection between these "limits at infinity" and horizontal asymptotes? Answer as completely and articulately as you can.