Worksheet - Techniques of Integration Necessary for Section 7.4

1. \( \int \frac{1}{2x-1} \, dx \).
   \textit{Hint: Let } u = 2x - 1. \textit{ Moral. You can integrate anything that looks like constant linear!}

2. \( \int \frac{2x-5}{(x^2-5x)} \, dx \).
   \textit{Hint: Let } u = x^2 - 5x. \textit{ Moral: Always check to see if you can use } u \text{-substitution before trying anything fancy!}

3. \( \int \frac{4x-1}{x^2+5} \, dx \).
   \textit{Hint: Split up the fraction, then use } u \text{-substitution (with } u = x^2 + 5) \text{ on one term and the following formula on the other:}
   \[ \int \frac{1}{x^2 + a^2} \, dx = \frac{1}{a} \tan^{-1} \left( \frac{x}{a} \right) + C. \]
   \textit{ Moral: You can integrate anything that looks like linear } \frac{1}{x^2+a^2} \text{!}

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4. \[ \int \frac{3x-1}{x^2+6x+11} \, dx. \]
   
   **Hint:** Complete the square in the denominator, \(i.e.\) \(x^2 + 6x + 11 = x^2 + 6x + 9 + 2 = (x + 3)^2 + 2.\) Then let \(u = x + 3,\) and apply the technique in problem 3, above.

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**Moral:** You can integrate anything that looks like \(\frac{\text{linear}}{\text{quadratic}}.\)

5. \[ \int \frac{x^3 - 3x^2 + 1}{x^2 + 1} \, dx. \]
   
   **Hint:** Perform polynomial division.
   
   Recall: to do long division we get the answer one digit at a time, then multiply, subtract, and get the remainder. Then the answer is \((\text{quotient}) + \frac{\text{remainder}}{\text{divisor}}).\)
   
   Example: 1650 divided by 38 is \(43\frac{16}{38}\).
   
   To do polynomial division, we do a very similar process with polynomials. Remember to write the terms in descending order by powers, and insert 0 coefficients for missing powers. In other words, the first step should look like
   
   \[
   x^2 + 0x + 1 \quad \longdiv{x^3 - 3x^2 + 0x + 1}
   \]

   Then the answer should be a polynomial (the quotient) plus a *proper* rational function (the remainder over \(x^2 + 1\)).

   Check to make sure you get \(x - 3 + \frac{-x + 4}{x^2 + 1}\). Integrate.

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**Moral:** When the integrand is an *improper* rational function, perform polynomial division to rewrite the quotient as a polynomial plus a *proper* rational function, then apply the previous techniques.

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