Math 76 Practice Problems for Midterm II
§§7.2-10.3

DISCLAIMER. This collection of practice problems is *not* guaranteed to be identical, in length or content, to the actual exam. You may expect to see problems on the test that are not exactly like problems you have seen before.

On the actual exam you will have more room to work the problems. You will see directions similar to these:

1. Please read directions carefully. Raise your hand if you are not sure what a problem is asking.
2. *You must explain your work thoroughly and unambiguously to receive full credit on questions or parts of questions designated as* ***Work and Answer****.*
3. **No calculators or notes are allowed on this exam.**
4. You have 65 minutes to complete your test, unless announced otherwise. Do not spend too long on any one problem. You do not have to do the problems in order. Do the easy ones first. Do not attempt the bonus question until you have completed the rest of the test. Before turning in your test, please make sure you have answered and double-checked all the questions.
5. If you need scratch paper, please raise your hand. You may not use your own paper. When you have finished your exam, please turn in any scratch paper you use.
6. Write your solutions in the space provided for each problem, or provide specific instructions as to where your work is to be found. *Make it clear what you want and don't want graded.* Your final answers should be boxed or circled.
7. Unless directed otherwise, only EXACT ANSWERS will receive full credit (i.e. $\sqrt{2}$, not 1.414).
8. In word problems, give units on all answers (e.g. feet, grams, gallons).
9. Don't stress! I'm rooting for you!

You will also be provided with the following information:

English system formulas:	Metric system formulas:
1 ft. = 12 in.	$F = m \cdot a$
5280 ft. = 1 mi.	$g = 9.8\,\text{m/s}^2$
16 oz. = 1 lb.	100 cm = 1 m
Weight of water: $\omega = 62.5\,\text{lb./ft}^3$	Weight of water: $\omega = 9800\,\text{N/m}^3$

Trigonometric identities:

$\sin^2 x + \cos^2 x = 1$	$\sin x \cos x = \frac{1}{2}\sin 2x$
$\tan^2 x + 1 = \sec^2 x$	$\sin A \cos B = \frac{1}{2}\left[\sin(A-B) + \sin(A+B)\right]$
$\sin^2 x = \frac{1}{2}(1 - \cos 2x)$	$\sin A \sin B = \frac{1}{2}\left[\cos(A-B) - \cos(A+B)\right]$
$\cos^2 x = \frac{1}{2}(1 + \cos 2x)$	$\cos A \cos B = \frac{1}{2}\left[\cos(A-B) + \cos(A+B)\right]$

General formulas:

$$F = \omega \int_a^b w(y)d(y)\; dy$$

$$\overline{x} = \frac{1}{A}\int_a^b x\left(f(x) - g(x)\right)\, dx \qquad \overline{y} = \frac{1}{A}\cdot\frac{1}{2}\int_a^b \left((f(x))^2 - (g(x))^2\right)\, dx$$

Multiple Choice. *Circle the letter of the best answer.*

1. $\displaystyle\int_{-\pi/4}^{\pi/4} \tan^2 x \, dx =$

   (a) $1 + \frac{\pi}{2}$

   (b) $1 - \frac{\pi}{4}$

   (c) $2 - \frac{\pi}{2}$

   (d) $2 + \frac{\pi}{4}$

2. The partial fraction decomposition of $\dfrac{4x^3 - 2x + 1}{(x^2+5)(x-3)^2}$ is

   (a) $\frac{A}{x^2+5} + \frac{B}{x-3} + \frac{C}{(x-3)^2}$

   (b) $\frac{Ax+B}{x^2+5} + \frac{C}{x-3} + \frac{D}{(x-3)^2}$

   (c) $\frac{Ax+B}{x^2+5} + \frac{C}{(x-3)^2}$

   (d) $\frac{Ax+B}{x^2+5} + \frac{C}{x-3} + \frac{Dx+E}{(x-3)^2}$

3. $\displaystyle\int \frac{2x-1}{(x+1)(x-2)} \, dx =$

   (a) $\ln|x+1| + \ln|x-2| + C$

   (b) $3\ln|x+1| - 2\ln|x-2| + C$

   (c) $\ln|x+1| - \ln|x-2| + C$

   (d) $-\ln|x+1| + \ln|x-2| + C$

4. $\displaystyle\int_0^e \ln x \, dx =$

   (a) 1

   (b) 0

   (c) ∞

   (d) $-\infty$

5. The length of the curve $x = y^3 - y$ from $y = 1$ to $y = 3$ is

   (a) $2\pi \int_1^3 \sqrt{1 + (3y^2-1)^2} \, dy$

   (b) $\int_1^3 \sqrt{9y^4 - 6y^2 + 2} \, dy$

   (c) $\int_1^3 \sqrt{1 + y^3 + y} \, dy$

   (d) $\int_1^3 \sqrt{3y^2} \, dy$

6. The area of the surface formed by rotating the curve $x = y^3 - y$ from $y = 1$ to $y = 3$ about the x-axis is

   (a) $2\pi \int_1^3 y\sqrt{1 + (3y^2-1)^2} \, dy$

   (b) $\int_1^3 \sqrt{9y^4 - 6y^2 + 2} \, dy$

   (c) $2\pi \int_1^3 (y^3 - y)\sqrt{1 + y^3 - y} \, dy$

   (d) $2\pi \int_1^3 y\sqrt{3y^2} \, dy$

7. A trough is filled with water. The ends of the trough are equilateral triangles with sides 8 m long and vertex at the bottom. The hydrostatic force on one end of the trough is

(a) $\dfrac{9800\sqrt{3}}{2}\displaystyle\int_0^{4\sqrt{3}} y(y-8)\,dy$

(b) $\dfrac{9800}{\sqrt{3}}\displaystyle\int_0^{4\sqrt{3}} y^2\,dy$

(c) $9800\displaystyle\int_0^{8} (8-y)y\,dy$

(d) $\dfrac{19600}{\sqrt{3}}\displaystyle\int_0^{4\sqrt{3}} (4\sqrt{3}-y)y\,dy$

8. The coordinates of the center of mass of the region enclosed by $y = x$, $y = x - 4$, $x = 1$ and $x = 3$ are

(a) $(2.5, 0.5)$

(b) $(1.75, 0)$

(c) $(2, 0)$

(d) $(2, 0.5)$

9. The equation of the line tangent to the curve $\begin{array}{c} x = e^{\sqrt{t}} \\ y = t - \ln t^2 \end{array}$ at the point corresponding to $t = 4$ is

(a) $y = \frac{2}{e^2}x + 4 - \ln 16$

(b) $y = \frac{1}{2}x + 4 - \ln 16 - \frac{1}{2}e^2$

(c) $y = \frac{e^2}{4}x + 4 - \ln 16 - \frac{1}{4}e^4$

(d) $y = \frac{2}{e^2}x + 2 - \ln 16$

10. In polar coordinates, the point $(3, \frac{\pi}{2})$ represents the same location as the point

(a) $(-3, -\frac{\pi}{2})$

(b) $(3, -\frac{\pi}{2})$

(c) $(-3, \frac{\pi}{2})$

(d) $(3, \frac{3\pi}{2})$

Fill-In.

1. $\displaystyle\int \sec^3 x\,dx =$ __________________________ .

2. $\displaystyle\int \sin^3 x\,dx =$ __________________________ .

3. To evaluate the integral $\displaystyle\int \sqrt{5+x^2}\,dx$, it is best to use the trigonometric substitution

$x =$ ________________ .
(function of θ)

4. $\displaystyle\int_1^{\infty} \frac{5}{x^3}\,dx =$ _____________ .

5. If $\dfrac{x^2-3}{(x^2+1)(x-2)} = \dfrac{Ax+B}{x^2+1} + \dfrac{C}{x-2}$, then

   (a) $A =$ ______________

   (b) $B =$ ______________

   (c) $C =$ ______________

6. The polar curve $r = \cos\theta$ is symmetric about the **(** x**-axis** | y**-axis** | **origin)** .

Graph. *More accuracy = more points!*

Let C be the curve $\begin{array}{c} x = \cos t \\ y = \sin t \cos t \end{array}$.

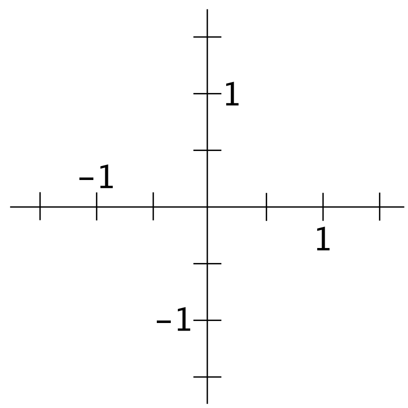
(a) Eliminate the parameter to find a Cartesian equation of C.

(b) Find the point(s) on the curve where the tangent is vertical.

(c) Find the point(s) on the curve where the tangent is horizontal.

(d) Find equation(s) of the tangent(s) to C at the point $(0, 0)$.

(e) Sketch a graph of C, labeling the features found in parts (b)-(d).



Work and Answer. *You must show all relevant work to receive full credit.*

1. Evaluate the integral $\displaystyle\int \cos^2 x \sin^4 x \, dx$.

2. Evaluate the integral $\displaystyle\int \tan^2 x \sec^4 x \, dx$.

3. Evaluate the integral $\displaystyle\int \tan^3 x \sec^3 x \, dx$.

4. Evaluate the integral $\displaystyle\int \tan^2 x \sec x \, dx$.

5. Evaluate the integral $\displaystyle\int \cos 2x \sin 3x \, dx$.

6. Evaluate the integral $\displaystyle\int \sqrt{4-9x^2} \, dx$.

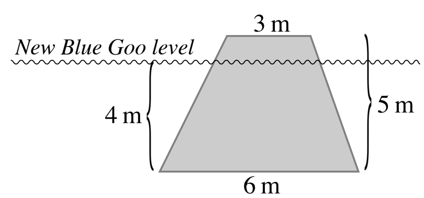
7. Evaluate the integral $\displaystyle\int \sqrt{4+9x^2} \, dx$.

8. Evaluate the integral $\displaystyle\int \sqrt{9x^2-4} \, dx$.

9. Find the length of the curve $f(x) = \frac{e^x+e^{-x}}{2}$ from $x = 0$ to $x = 1$.

10. Find the area of the surface formed by rotating the curve $f(x) = \frac{e^x+e^{-x}}{2}$ from $x = 0$ to $x = 1$ about the x-axis.

11. Find the hydrostatic force on the wall shown. The fluid is New Blue Goo (density 1500 kg/m^3).



12. For the curve $\begin{array}{c} x = 1 + \tan t \\ y = \cos 2t \end{array}$, find $\frac{dy}{dx}$ in terms of t.

13. Find the point(s) on the polar curve $r = e^\theta$ where the tangent is

    (a) horizontal

    (b) vertical.

Some kind of **BONUS.**