Math 75 Practice Problems for Midterm III
§§4.2-5.4

DISCLAIMER. This collection of practice problems is *not* guaranteed to be identical, in length or content, to the actual exam. You may expect to see problems on the test that are not exactly like problems you have seen before.

On the actual exam you will have more room to work the problems. You will see directions similar to these:

1. Please read directions carefully. Raise your hand if you are not sure what a problem is asking.
2. *You must explain your work thoroughly and unambiguously to receive full credit on questions or parts of questions designated as* ***Work and Answer****.*
3. **No calculators or notes are allowed on this exam.**
4. You have 65 minutes to complete your test, unless announced otherwise. Do not spend too long on any one problem. You do not have to do the problems in order. Do the easy ones first. Do not attempt the bonus question until you have completed the rest of the test. Before turning in your test, please make sure you have answered and double-checked all the questions.
5. If you need scratch paper, please raise your hand. You may not use your own paper. When you have finished your exam, please turn in any scratch paper you use.
6. Write your solutions in the space provided for each problem, or provide specific instructions as to where your work is to be found. *Make it clear what you want and don't want graded.* Your final answers should be boxed or circled.
7. Unless directed otherwise, only EXACT ANSWERS will receive full credit (i.e. $\sqrt{2}$, not 1.414).
8. In word problems, give units on all answers (e.g. feet, grams, gallons).
9. Don't stress! I'm rooting for you!

True or False. Circle **T** if the statement is *always* true; otherwise circle **F**.

1. If $f(x)$ is a continuous function and $f(3) = 2$ and $f(5) = -1$, then $f(x)$ has a root between 3 and 5. **T** **F**

2. The function $g(x) = 2x^3 - 12x + 5$ has 5 real roots. **T** **F**

3. If $h(x)$ is a continuous function and $h(1) = 4$ and $h(2) = 5$, then $h(x)$ has no roots between 1 and 2. **T** **F**

4. The only x-intercept of $f(x) = x^3 - x^2 + 2x - 2$ is $(1, 0)$. *(Challenge problem!)* **T** **F**

5. If the velocity of an object at time t is $v(t) = 4t^2 + 1$ ft./s, then its distance in feet at time t is $s(t) = \frac{4}{3}t^3 + t$. **T** **F**

6. The function $F(x) = \sin 2x + 52$ is an antiderivative of the function $f(x) = 2\cos 2x$. **T** **F**

7. The function $G(x) = 4x^3$ is an antiderivative of the function $g(x) = x^4 - 2$. **T** **F**

8. $-1 + 0 + \frac{1}{3} + \frac{1}{2} + \frac{3}{5} + \frac{2}{3} = \sum_{i=-1}^{4} \frac{i}{i+2}$. **T** **F**

9. $\sum_{i=2}^{4} \frac{i^2}{2} = \frac{29}{2}$. **T** **F**

10. If $g(x)$ is an odd function which is continuous on the interval $[-3, 3]$, then $\int_{-3}^{3} g(x)\, dx = 0$. **T** **F**

11. If $h(x)$ is an even function which is continuous on the interval $[-3, 3]$, then $\int_{-3}^{3} h(x)\, dx = 2\int_{0}^{3} h(x)\, dx$. **T** **F**

Multiple Choice. *Circle the letter of the best answer.*

1. If $x_1 = 1$ is a first approximation of a solution to the equation $x^4 = 6 - 3x$, then using Newton's Method the second approximation is $x_2 =$

   (a) $\frac{9}{7}$ (c) $\frac{9}{2}$

   (b) $\frac{5}{7}$ (d) $-\frac{5}{2}$

2. $\int_{-2}^{2} \sqrt{4 - x^2}\, dx =$

   (a) $-\frac{1}{6}$ (c) 2π

   (b) 0 (d) does not exist.

3. $\int_{0}^{\pi/4} \sec x \tan x\, dx =$

   (a) $\sqrt{2} - 1$ (c) $1 - \frac{\sqrt{2}}{2}$

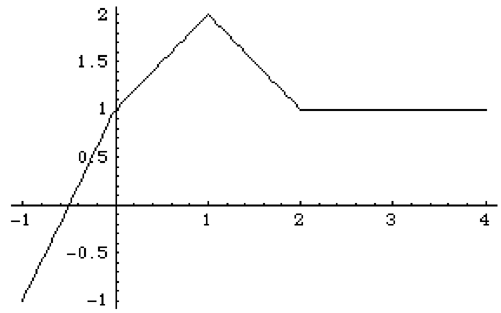
   (b) $\sqrt{2}$ (d) does not exist.

4. If $f(x)$ is continuous, then $\int_{0}^{\pi/4} f(x)\, dx - \int_{0}^{\pi} f(x)\, dx =$

   (a) $\int_{\pi/4}^{\pi} f(x)\, dx$ (c) $\int_{0}^{3\pi/4} f(x)\, dx$

   (b) $\int_{\pi}^{\pi/4} f(x)\, dx$ (d) $\int_{\pi}^{0} f(x)\, dx$

For #5-6, use the graph of $f(x)$ shown below to answer the questions.



5. $\displaystyle\int_{-1}^{0} f(x)\, dx \ =$

(a) -1

(b) 0

(c) 1

(d) 2

6. $\displaystyle\int_{2}^{1} f(x)\, dx \ =$

(a) $-\dfrac{3}{2}$

(b) $\dfrac{3}{2}$

(c) -1

(d) 1

Fill-In. *If an answer is undefined, write "D.N.E."*

1. If a polynomial function $f(x)$ has 3 solutions to the equation $f'(x) = 0$, then $f(x)$ has at most ________ roots.

2. $\displaystyle\int (\sqrt[3]{x} - \sec^2 x)\, dx =$ _______________ .

3. $\displaystyle\int_{-1}^{2} \sqrt[4]{x}\, dx =$ _______________ .

4. $\displaystyle\int_{-1}^{2} \sqrt[3]{x}\, dx =$ _______________ .

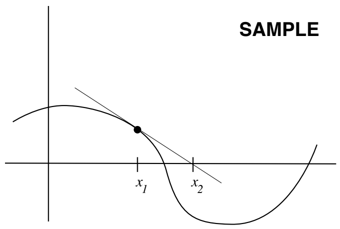
5. If $F(x) = \displaystyle\int_{5}^{x} \sqrt{5t - t^4}\, dt$, then $F'(x) =$ _______________ .

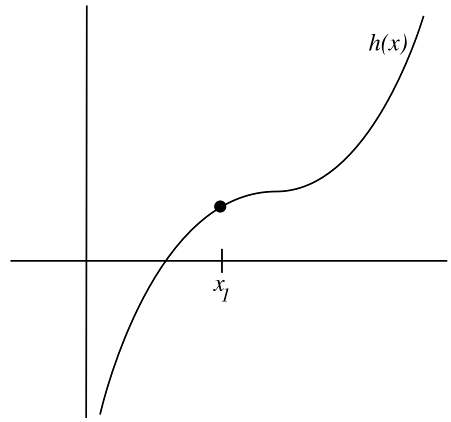
6. If $G(x) = \displaystyle\int_{x}^{\sqrt{\pi}} \cot(6t^2)\, dt$, then $G'(x) =$ _______________ .

7. If the interval $[-4, 7]$ is divided into 6 equal subintervals, then the width of each subinterval is __________ .

8. $\displaystyle\int_{-2}^{1} |x|\, dx =$ _______________ .

Graph.
For the function $h(x)$ graphed at right and the initial guess x_1 shown, draw tangent lines to determine x_2 and x_3 according to Newton's Method, as in the sample (the sample is only completed through x_2). Label x_2 and x_3 on the x-axis.





Work and Answer. *You must show all relevant work to receive full credit.*

1. Prove that the function $f(x) = -x^3 - 6x + 1$ has exactly one real root by completing the following:

   (a) Use the Intermediate Value Theorem to show that the function $f(x) = -x^3 - 6x + 1$ has *at least* one real root.

   (b) Use Rolle's Theorem to show that the function $f(x) = -x^3 - 6x + 1$ has *at most* one real root.

2. Estimate the root of $f(x) = x^3 + 2x - 1$ using two iterations of Newton's Method (*i.e.* compute x_3) with the initial guess $x_1 = 0$. *Express your answer as an exact fraction.*

3. Evaluate $\displaystyle\int \frac{2}{t^3}\, dt$.

4. Evaluate $\displaystyle\int \frac{2}{1+x^2}\, dx$.

5. (a) Estimate $\int_0^{\pi} \sin\theta \, d\theta$ using 3 rectangles and midpoints.

(b) Evaluate $\int_0^{\pi} \sin\theta \, d\theta$ exactly.

(c) What is the error of the estimate you made in part (a)?

6. If $F(x) = \int_5^{\sin^2 x} (3t - 5) \, dt$,

(a) Evaluate $F'(x)$.

(b) Evaluate $F(x)$.

(c) Show that the derivative of the function you obtained in (b) equals the function you obtained in (a).

7. Evaluate $\int_0^{\pi/3} x^2 - \sin x \, dx$.

8. An object travels in a straight line with velocity function $v(t) = \frac{3}{t} - 4e^t$ feet per second. Determine the net change in position (in feet) over the time interval $2 \leq t \leq 5$.

Some kind of **BONUS.**