Math 75B Practice Midterm I
Ch. 9-11 (Ebersole), 3.6-3.8, 3.10, 4.1 (Stewart)

DISCLAIMER. This collection of practice problems is *not* guaranteed to be identical, in length or content, to the actual exam. You may expect to see problems on the test that are not exactly like problems you have seen before.

On the actual exam you will see directions similar to these:

1. Please read directions carefully. Raise your hand if you are not sure what a problem is asking.
2. *You must explain your work thoroughly and unambiguously to receive full credit on questions or parts of questions designated as **Work and Answer**.*
3. **No calculators or notes are allowed on this exam.**
4. You have 50 minutes to complete your test, unless announced otherwise. Do not spend too long on any one problem. You do not have to do the problems in order. Do the easy ones first. Do not attempt the bonus question until you have completed the rest of the test. Before turning in your test, please make sure you have answered and double-checked all the questions.
5. If you need scratch paper, please raise your hand. You may not use your own paper. When you have finished your exam, please turn in any scratch paper you use.
6. Write your solutions in the space provided for each problem, or provide specific instructions as to where your work is to be found. *Make it clear what you want and don't want graded.* Your final answers should be boxed or circled.
7. Don't stress! I'm rooting for you!

True or False. Circle **T** if the statement is *always* true; otherwise circle **F**.

1. If $f(x) = 3x^4 - 2x + 1$, then $f''(x) = 12x^3 - 2$. **T** **F**
2. If $g(x) = x^x$, then $g'(x) = x \cdot x^{x-1}$. **T** **F**
3. $\dfrac{d}{dt} \cos^{-1}(3t^2 + 1) = -\dfrac{6t}{\sqrt{1 - (3t^2+1)^2}}$. **T** **F**
4. $\dfrac{d}{dt} \tan^{-1}(3t^2 + 1) = \dfrac{6t}{\sqrt{1 - (3t^2+1)^2}}$. **T** **F**
5. The range of the function $f(x) = \sin^{-1} x$ is $\left[-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right]$. **T** **F**
6. The range of the function $f(x) = \cos^{-1} x$ is $\left[-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right]$. **T** **F**
7. The range of the function $f(x) = \tan^{-1} x$ is $\left[-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right]$. **T** **F**

8. $\dfrac{d}{dt}(\log(3t^2+1)) = \dfrac{(6\ln 10)t}{3t^2+1}$. **T** **F**

9. If $3x^2y = \tan(y^2)$, then $\dfrac{dy}{dx} = \dfrac{-6xy}{3x^2 - 2y\sec^2(y^2)}$. **T** **F**

10. The absolute maximum value of $f(x) = \dfrac{1}{x}$ on the interval $[2, 4]$ is 2. **T** **F**

Multiple Choice. *Circle the letter of the best answer.*

1. If $f(x) = \tan x$, then $f''(x) =$

(a) $\dfrac{2\sin x}{\cos^3 x}$ (c) $\sec^2 x$

(b) $\dfrac{1}{\sin^2 x}$ (d) $2\sec x \tan x$

2. The 2004$^{\text{th}}$ derivative of $f(x) = \sin x$ is

(a) $\sin x$ (c) $-\sin x$

(b) $\cos x$ (d) $-\cos x$

3. If $x^2 - y^2 = 4$, then $\dfrac{dy}{dx} =$

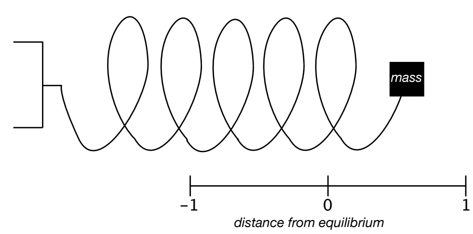
(a) $\dfrac{y}{x}$ (c) $\dfrac{x}{y}$

(b) $-\dfrac{y}{x}$ (d) $-\dfrac{x}{y}$

4. A mass attached to the end of a spring is pulled and then released. t seconds after release, the distance of the mass from equilibrium is $s(t) = \cos 2\pi t$ centimeters. The acceleration of the mass after 3 seconds is

(a) 0 cm/s^2 (c) $-4\pi^2$ cm/s^2

(b) -4π cm/s^2 (d) -2π cm/s^2



5. The absolute minimum of $f(x) = -x^2 + 6x + 1$ on the interval $[0, 5]$ is at $x =$

(a) 0 (c) 2

(b) 1 (d) 3

Fill-In.

1. $\sin^{-1}(1) =$ ____________

2. $\cos^{-1}(1) =$ ____________

3. $\sin^{-1}(\cos 3\pi) =$ ____________

4. $\cos^{-1}\left(\tan \frac{\pi}{4}\right) =$ ____________

5. $\tan\left(\cos^{-1} \frac{2}{7}\right) =$ ____________

6. $\sin\left(\sin^{-1} \frac{5}{2}\right) =$ ____________

7. $\sin^{-1}\left(\sin \frac{3\pi}{2}\right) =$ ____________

8. The absolute maximum value of the function $g(x) = \dfrac{3}{x-5}$ on the interval $[-3, -1]$ is ____________ .

Work and Answer. *You must show all relevant work to receive full credit.*

1. If $y \tan y = 3t - \dfrac{y}{t}$, find $\dfrac{dy}{dt}$.

2. Find the slope of the tangent line to the graph of $\dfrac{(x+2)^2}{9} + \dfrac{y^2}{4} = 1$ at the point $(-2, 2)$.

3. If a stone is thrown vertically upward from the surface of the moon with a velocity of 10 m/s, then its height (in meters) after t seconds is $s(t) = 10t - 0.83t^2$.
   (a) What is the velocity of the stone after 3 seconds?
   (b) What is the acceleration of the stone after 3 seconds?
   (c) When does the stone reach its maximum height?

4. Find the 2006th derivative of the function $f(x) = \cos(2007x)$.

5. Find the derivative of the function $g(x) = (\sin x)^{3x+1}$.

6. Find the derivative of the function $h(x) = \dfrac{(3x^2-4)^{10}\cos(4x)}{e^x(59x^3+8x)^{25}}$.

7. A spotlight on the ground shines on a wall 15 m away. If a man 2 m tall walks from the spotlight toward the wall at a speed of 1.8 m/s, how fast is the length of his shadow on the wall decreasing when he is 3 m from the building?

8. (a) Find the x-values where the function $f(x) = 4x^3 - 7x^2 + 1$ attains its absolute maximum and absolute minimum on the interval $[-1, 1]$.
   (b) Find the absolute maximum and the absolute minimum values of $f(x)$ on the above interval.

Some kind of **BONUS.**