Math 111 Practice Midterm I
Ch. 1, 2

DISCLAIMER. This collection of practice problems is *not* guaranteed to be identical, in length or content, to the actual exam. You may expect to see problems on the test that are not exactly like problems you have seen before.

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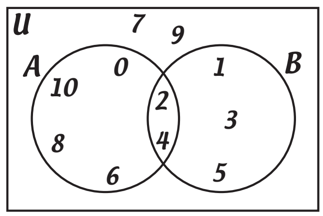
1. Review all terms and notations from chapters 0-2.

2. Let $U = \{x \in \mathbb{Z} \mid 0 \le x \le 10\}$ be the universal set, $A = \{x \in U \mid x \text{ is even}\}$, $B = \{1, 2, 3, 4, 5\}$.

   (a) Draw a Venn diagram that illustrates the above sets.



   (b) Determine (i.e. list all the elements of) the following sets: $A \cap B$, $\overline{A}$, $A \cup \overline{B}$.

   $A \cap B = \{2, 4\}$, $\overline{A} = \{x \in U \mid x \text{ is odd}\}$, $A \cup \overline{B} = \{0, 2, 4, 6, 7, 8, 9, 10\}$.

   (c) How many elements does $A \times B$ have?

   Since $|A| = 6$, and $|B| = 5$, $|A \times B| = 30$.

   (d) List three of the elements of $A \times B$.

   $(4, 5)$, $(8, 3)$, $(6, 3)$. (answers vary)

3. Let $A = \{1\}$, $B = \{2\}$, $C = \{\{3\}\}$, $D = \{1, \{2\}, \{1, 2, 3\}\}$.

   (a) Which of the following statements are true: $A \in D$, $A \subseteq D$, $B \in D$, $B \subseteq D$, $C \in D$, $C \subseteq D$, $\emptyset \in D$, $\emptyset \subseteq D$?

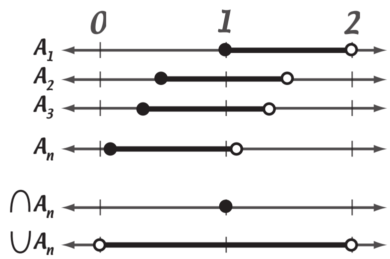
   $A \subseteq D$, $B \in D$, and $\emptyset \subseteq D$ are true. The rest are false.

   (b) What are the cardinalities of these four sets?

   $|A| = 1$, $|B| = 1$, $|C| = 1$, $|D| = 3$.

4. For each $n \in \mathbb{N}$ let $A_n = \left[\frac{1}{n}, \frac{n+1}{n}\right)$. Determine $\bigcup_{n=1}^{\infty} A_n$ and $\bigcap_{n=1}^{\infty} A_n$. (No formal proof is required, but please provide an explanation of your answer; a picture might be helpful.)

Here is a number line showing A_n for a few n, along with $\bigcup_{n=1}^{\infty} A_n$ and $\bigcap_{n=1}^{\infty} A_n$:



From the picture, we can see that $\bigcup_{n=1}^{\infty} A_n = (0, 2)$ and $\bigcap_{n=1}^{\infty} A_n = \{1\}$.

5. Let P and Q be statements.

   (a) Show that $P \iff Q$ and $(P \wedge Q) \vee ((\sim P) \wedge (\sim Q))$ are logically equivalent.

   This compound statement is plausible, since $P \iff Q$ means that either P and Q are both true or they are both false. To prove it more formally we construct a truth table showing $P \iff Q$ and $(P \wedge Q) \vee ((\sim P) \wedge (\sim Q))$:

P	Q	$\sim P$	$\sim Q$	$P \iff Q$	$P \wedge Q$	$(\sim P) \wedge (\sim Q)$	$(P \wedge Q) \vee ((\sim P) \wedge (\sim Q))$
T	T	F	F	**T**	T	F	**T**
T	F	F	T	**F**	F	F	**F**
F	T	T	F	**F**	F	F	**F**
F	F	T	T	**T**	F	T	**T**

   Since the two columns are the same, we know the two statements are logically equivalent.

   (b) The compound statement $(P \iff Q) \iff ((P \wedge Q) \vee ((\sim P) \wedge (\sim Q)))$ is an example of a <u>tautology</u>.

   (c) The compound statement $(P \iff Q) \iff \sim ((P \wedge Q) \vee ((\sim P) \wedge (\sim Q)))$ is an example of a <u>contradiction</u>.

6. Let

$$\begin{aligned} P(x) &: x \text{ is wearing shoes.} \\ Q(x) &: x \text{ has an umbrella.} \\ R(x) &: x \text{ walks to class.} \end{aligned}$$

   where x belongs to the set $S = \{\text{students at Fresno State}\}$.

   (a) Write the following statements in words:

   - $(P(\text{Jasper}) \wedge Q(\text{Jasper})) \Rightarrow R(\text{Jasper})$.

- $\sim Q(\text{Cameron}) \Rightarrow \sim R(\text{Cameron})$.
- $R(\text{Dirk}) \iff P(\text{Yolanda})$.
- $\forall x, P(x) \wedge Q(x)$.
- $\exists x, \sim (R(x) \Rightarrow P(x))$.

In order, we have

- If Jasper is wearing shoes and has an umbrella, then he walks to class.
- If Cameron does not have an umbrella, then he does not walk to class.
- Dirk walks to class if and only if Yolanda is wearing shoes.
- All students at Fresno State wear shoes and have an umbrella.
- There is a student at Fresno State who walks to class and does not wear shoes.

(The phrasing may vary.)

(b) Determine the negations of the above statements. Write them in words and in symbols.

In order, we have, in words,

- Sometimes Jasper is wearing shoes and has an umbrella and does not walk to class.
- Sometimes Cameron does not have an umbrella, and he walks to class.
- Sometimes, either Dirk walks to class and Yolanda is not wearing shoes, or Yolanda is wearing shoes and Dirk does not walk to class.
- There is a student at Fresno State who either does not wear shoes or does not have an umbrella.
- All students at Fresno State wear shoes if they walk to class.

(The phrasing may vary.)

In symbols, we have

- $(P(\text{Jasper}) \wedge Q(\text{Jasper})) \wedge \sim R(\text{Jasper})$.
- $\sim Q(\text{Cameron}) \wedge R(\text{Cameron})$.
- $(R(\text{Dirk}) \wedge (\sim P(\text{Yolanda}))) \vee (P(\text{Yolanda}) \wedge (\sim R(\text{Dirk})))$.
- $\exists x, (\sim P(x)) \vee (\sim Q(x))$.
- $\forall x, R(x) \Rightarrow P(x)$.