Math 75B Practice Final §§2.6-5.5 (Stewart)

DISCLAIMER. This collection of practice problems is *not* guaranteed to be identical, in length or content, to the actual exam. You may expect to see problems on the test that are not exactly like problems you have seen before.

On the actual exam you will have more room to work the problems. You will see directions similar to these:

1. Please read directions carefully. Raise your hand if you are not sure what a problem is asking.
2. *You must explain your work thoroughly and unambiguously to receive full credit on questions or parts of questions designated as* ***Work and Answer****.*
3. **No calculators or notes are allowed on this exam.**
4. You have 2 hours to complete your test, unless announced otherwise. Do not spend too long on any one problem. You do not have to do the problems in order. Do the easy ones first. Do not attempt the bonus question until you have completed the rest of the test. Before turning in your test, please make sure you have answered and double-checked all the questions.
5. If you need scratch paper, please raise your hand. You may not use your own paper. When you have finished your exam, please turn in any scratch paper you use.
6. For **Work and Answer** problems, write your solutions in the space provided for each problem, or provide specific instructions as to where your work is to be found. *Make it clear what you want and don't want graded.* Your final answers should be boxed or circled.
7. Don't stress! I'm rooting for you!

Multiple Choice. *Circle the letter of the best answer.*

1. $\int (\sin x + 2\cos x)\, dx =$

   (a) $-\cos x + 2\sin x + C$

   (b) $-\cos x - 2\sin x + C$

   (c) $\cos x + 2\sin x + C$

   (d) $\cos x - 2\sin x$

   (e) $-\cos x - 2\sin x$

2. If $\sin y + xy = 2x$, then $\dfrac{dy}{dx} =$

   (a) $\cos 2y$

   (b) $\dfrac{2-y}{\cos y + x}$

   (c) $\dfrac{\cos y}{2xy}$

   (d) $\dfrac{2x+y}{\cos y}$

   (e) $y + 2x\cos y$

3. If $3x\frac{dy}{dx} - 4 = \cos y \cdot \frac{dy}{dx}$, then $\frac{dy}{dx} =$

(a) $\dfrac{3x}{4 - \cos y}$

(b) $\dfrac{3x - 4}{\cos y}$

(c) $\dfrac{4 + \cos y}{3x}$

(d) $\dfrac{\cos y}{3x - 4}$

(e) $\dfrac{4}{3x - \cos y}$

4. If $f(x) = \ln(7x^3 + 1)$, then $f'(x) =$

(a) $\dfrac{1}{7x^3 + 1}$

(b) $\dfrac{21x^2}{7x^3 + 1}$

(c) $3\ln(7x + 1)$

(d) $\dfrac{1}{21x^2}$

(e) $\dfrac{7}{7x^3 - 21x^2}$

5. Suppose you know that $f'(x) = g(x)$. Which of the following must be true?

(a) $\int g(x)\, dx = f(x)$

(b) $\int g(x)\, dx = f(x) + C$

(c) $\dfrac{d}{dx}(g(x)) = f(x)$

(d) $\dfrac{d}{dx}(g(x)) = f(x) + C$

(e) All of the above are true.

6. If $y = \int_0^{x^2} \tan t\, dt$, then $y' =$

(a) $2x\tan(x^2)$

(b) $\tan(x^2)$

(c) $\tan x$

(d) $2x\sec^2(x^2)$

(e) $\sec^2(x^2)$

7. The inflection point(s) of the function $y = 3x^5 - 5x^4 + 60x - 60$ is/are

(a) $(0, -60)$ only

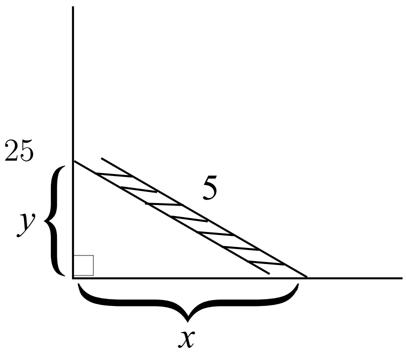
(b) $(-1, -128)$ only

(c) $(-1, -128)$ and $(1, -2)$ only

(d) $(1, -2)$ only

(e) $(0, -60)$, $(1, -2)$, and $(-1, -128)$ only

8. A ladder 5 meters long, leaning against a wall, begins to slide. According to the diagram at right,



(a) $\dfrac{dx}{dt} = -\dfrac{dy}{dt}$

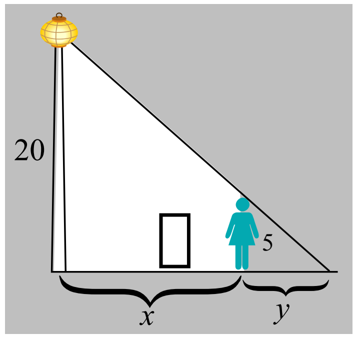
(b) $2x\dfrac{dy}{dx} + 2y\dfrac{dx}{dy} = 0$

(c) $x\dfrac{dx}{dt} = -y\dfrac{dy}{dt}$

(d) $\left(\dfrac{dx}{dt}\right)^2 + \left(\dfrac{dy}{dt}\right)^2 = 25$

(e) $2x + 2y = 0$

9. A woman 5 ft. tall is walking toward a streetlight 20 ft. tall. According to the diagram at right, $\dfrac{dy}{dt}$ is



(a) the length of her shadow, a positive number for all t

(b) the rate at which the length of her shadow is changing, a positive number for all t

(c) the rate at which the length of her shadow is changing, a negative number for all t

(d) her speed, a positive number for all t

(e) her speed, a negative number for all t

10. Which of the following is the linear approximation of the function $f(x) = \sqrt[3]{x}$ near the number $a = 1$?

(a) $y = \dfrac{1}{3}x + 1$

(b) $y = \dfrac{1}{3}x + \dfrac{2}{3}$

(c) $y = x - \dfrac{2}{3}$

(d) $y = x + 3$

(e) $y = 3x + 2$

11. $\displaystyle\int_0^4 |x - 3|\, dx =$

(a) 24

(b) 2

(c) 4

(d) 20

(e) 5

Fill-In.

1. The vertical asymptote(s) for the function $f(x) = \dfrac{x}{x^2-1}$ is/are ________________________ and the horizontal asymptote(s) is/are ________________________ .

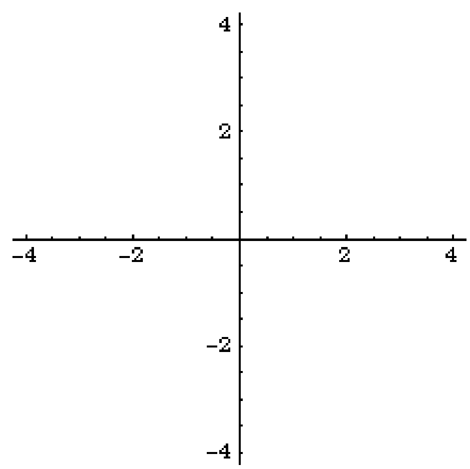
2. The graph of the function $f(x) = x^4 + 2x^3$ is increasing on the interval(s) ____________ ________________________ .

3. According to Rolle's Theorem, the maximum number of real roots of the function $f(x) = 4x^5 + 2x - 3$ is _________ .

4. Given the initial guess $x_1 = 2$, the second approximation to a root of $g(x) = x^3 - 4x - 1$ using Newton's Method is $x_2 =$ _________ .

Graphs. *More accuracy = more points!*

1. For the function $f(x) = \frac{1}{3}x^3 - 2x$,



(a) find the critical **points** and intervals of increase/decrease

(b) find the inflection **points** and intervals of concave up/concave down

(c) discuss any symmetry $f(x)$ may or may not have

(d) find the equations of any vertical and/or horizontal asymptotes

(e) find the y-intercept

(f) On the axes at right, sketch an accurate graph of $f(x)$.

Work and Answer. *You must show all relevant work to receive full credit.*

1. Find the slope of the tangent line to the curve $x^2 + xy + y^3 = 7$ at the point $(2, 1)$.

2. Use logarithmic differentiation to find the derivative of the function $f(x) = x^{\ln x}$.

3. The area of a circular oil spill is increasing at the constant rate of 50π m^2 per minute. How fast is the radius of the spill increasing when the radius is 5 m?

   Simplify your answer and give units (e.g. grams, miles per gallon, etc.). You may use the fact that the area of a circle of radius r is $A = \pi r^2$.

4. Two cars start moving from the same point. One travels north at 40 mi/h and the other travels east at 30 mi/h. At what rate is the distance between the cars increasing two hours later? *Simplify your answer and give units (e.g. feet, kilograms, etc.).*

5. A farmer has 400 meters of fencing with which to fence 3 sides of a rectangular horse corral. What is the maximum area she can enclose?

6. Evaluate $\int_{-1}^{2} (x^2 + 2)\, dx$.

7. Evaluate $\int x(3x^2 + 1)^5\, dx$.

8. Evaluate $\int_{0}^{1} x\cos(x^2 + 1)\, dx$.

Some kind of **BONUS.**