

Math 75B Selected Homework Solutions

12-A #1, 2

12-B #4

2.8 #11, 14, 20, 22, 28

Completeness: 16 (2 points each)

Format: 10

Total: 26 points

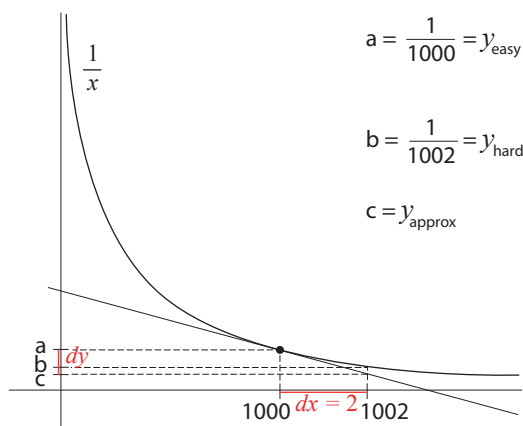
§2.8 #14. Use a linear approximation (or differentials) to estimate $\frac{1}{1002}$.

The number 1002 is being plugged into the function $f(x) = \frac{1}{x}$. An “easy” number close to 1002 to plug in is $x_{\text{easy}} = 1000$. We have

$$x_{\text{hard}} = 1002 \qquad y_{\text{hard}} = \frac{1}{1002} \approx ??$$

$$x_{\text{easy}} = 1000 \qquad y_{\text{easy}} = \frac{1}{1000} = 0.001$$

$$dx = 1002 - 1000 = 2$$



$$a = \frac{1}{1000} = y_{\text{easy}}$$

$$b = \frac{1}{1002} = y_{\text{hard}}$$

$$c = y_{\text{approx}}$$

Solution 1. (*tangent line approximation*)

First we find the equation of the tangent line to $f(x)$ at $x = 1000$, as shown. We have $f'(x) = -\frac{1}{x^2}$, so the slope of the line is $f'(1000) = -\frac{1}{(1000)^2} = -\frac{1}{1,000,000} = -0.000001$.

Now we plug in $(x_{\text{easy}}, y_{\text{easy}})$ and the above slope to $y = mx + b$ and solve for b :

$$\begin{aligned} y &= mx + b \\ 0.001 &= -0.000001(1000) + b \\ b &= 0.001 + 0.001 = 0.002 \\ y &= -0.000001x + 0.002 \end{aligned}$$

Now we plug in $x_{\text{hard}} = 1002$ to get y_{approx} (see the picture):

$$\begin{aligned} y_{\text{approx}} &= -0.000001(1002) + 0.002 \\ &= -\frac{1002}{1000000} + \frac{2}{1000} \\ &= \frac{-1002 + 2 \cdot 1000}{1000000} = \frac{998}{1000000} = \boxed{0.000998} \end{aligned}$$

Solution 2. (*differentials*)

We have

$$\begin{aligned}y &= \frac{1}{x} \\ \frac{dy}{dx} &= -\frac{1}{x^2} \\ dy &= -\frac{1}{x^2} dx\end{aligned}$$

Plugging in $dx = 2$ and $x = x_{\text{easy}} = 1000$, we get

$$dy = -\frac{1}{1000^2} \cdot 2 = -\frac{2}{1000000} = -0.000002.$$

Finally, to get y_{approx} we take $y_{\text{easy}} + dy$:

$$y_{\text{easy}} + dy = 0.001 - 0.000002 = \boxed{0.000988}$$

As a reality check, this number is a little bit less than $\frac{1}{1000} = 0.001$, as expected. Using a calculator, we can check the accuracy: $\frac{1}{1002}$ is approximately 0.000998004.