Math 75B Selected Homework Solutions

4.2 #2, 7, 13, 18, 20
4.6 #1, 4, 10, 12, 19, 29*

Completeness:	10	
Format:	10	
Total:	20	points
	(+2	possible bonus points)

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§4.2 #18. Show that the function $f(x) = 2x - 1 - \sin x$ has exactly one real root.

Strategy: we will use the Intermediate Value Theorem (IVT) to show that $f(x)$ has *at least one* real root. Then we will use Rolle's Theorem to show that $f(x)$ has *at most one* real root.

First of all, $f(x)$ is continuous and differentiable everywhere, so both of the above theorems apply.

We have $f(0) = -1 < 0$ and $f\left(\frac{3\pi}{2}\right) = 2\left(\frac{3\pi}{2}\right) - 1 - (-1) = 3\pi > 0$, so by IVT there must be at least one root of $f(x)$ between 0 and $\frac{3\pi}{2}$.

Now using Rolle's Theorem, we have $f'(x) = 2 - \cos x \overset{\text{set}}{=} 0$ implies $\cos x = 2$, which is impossible. So there is no x for which $f'(x) = 0$. Therefore by Rolle's Theorem $f(x)$ has at most one real root.

Since $f(x)$ has at least one real root, and also has at most one real root, we conclude that $f(x)$ has *exactly one* real root. □

§4.6 #10. Use Newton's Method to approximate $\sqrt[7]{1000}$ to 8 decimal places.

Note that $\sqrt[7]{1000}$ is a root of the function $f(x) = x^7 - 1000$, so we can use Newton's Method to approximate this root.

Since $2^7 = 128$ and $3^7 = 2187$, we know that $\sqrt[7]{1000}$ is between 2 and 3. After experimenting with a calculator, I found that $(2.7)^7 \approx 1046.03532$, so I decided to let $x_1 = 2.7$. You may choose a different x_1, as long as it is between 2 and 3 and you save all the digits in your calculator — though you may need more iterations if your x_1 is further away.

We have $f'(x) = 7x^6$; using the formula $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$, I get the numbers in the table below:

n	x_n	$f(x_n)$	$f'(x_n)$
1	2.7	46.03532	2711.943423
2	2.68302497	0.85923698	2611.237218
3	2.682695916	0.00031607	2609.316311
4	2.682695795		

At this point, $f(x_4) = f(2.682695795) = 0$, according to my calculator. Sure enough, my calculator says $\sqrt[7]{1000} = 2.682695795$, which is not *exactly* true but is true to the number of decimal places my calculator keeps track of, which is 9. So, to 8 decimal places,

$$\boxed{\sqrt[7]{1000} \approx 2.68269580}$$