Worksheet – Techniques of Integration Necessary for Section 6.3

1. $\displaystyle\int \frac{1}{2x-1}\,dx.$

   Hint. Let $u = 2x - 1$.

   Moral. You can integrate anything that looks like $\dfrac{\text{constant}}{\text{linear}}$!

2. $\displaystyle\int \frac{2x-5}{(x^2-5x)^3}\,dx.$

   Hint. Let $u = x^2 - 5x$.

   Moral. Always check to see if you can use u-substitution before trying anything fancy!

3. $\displaystyle\int \frac{4x-1}{x^2+5}\,dx.$

   Hint. Split up the fraction, then use u-substitution (with $u = x^2 + 5$) on one term and the following **formula** on the other:

$$\int \frac{1}{x^2+a^2}\,dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C.$$

   Moral: You can integrate anything that looks like $\dfrac{\text{linear}}{x^2+a^2}$!

4. $\displaystyle\int \frac{3x-1}{x^2+6x+11}\,dx.$

Hint. Complete the square in the denominator, *i.e.* $x^2+6x+11 = x^2+6x+9+2 = (x+3)^2+2$. Then let $u = x+3$, and apply the technique in problem 3, above.

Moral: You can integrate anything that looks like $\dfrac{\text{linear}}{\text{quadratic}}$!

5. $\displaystyle\int \frac{x^3-3x^2+1}{x^2+1}\,dx.$

Hint. Perform **polynomial division**.

> Recall: to do long division we get the answer one digit at a time, then multiply, subtract, and get the remainder. Then the answer is (quotient) + $\dfrac{\text{remainder}}{\text{divisor}}$.
> Example: 1650 divided by 38 is $43\frac{16}{38}$.
> To do polynomial division, we do a very similar process with polynomials. Remember to write the terms in descending order by powers, and insert 0 coefficients for missing powers. In other words, the first step should look like
>
> $$x^2+0x+1 \;\Big|\; \overline{x^3-3x^2+0x+1}$$
>
> Then the answer should be a polynomial (the quotient) plus a *proper* rational function (the remainder over x^2+1).

Check to make sure you get $x-3+\dfrac{-x+4}{x^2+1}$. Integrate.

Moral. When the integrand is an *improper* rational function, perform polynomial division to rewrite the quotient as a polynomial plus a *proper* rational function, then apply the previous techniques.