Section 6.2 - Trigonometric Integrals Worksheet

Recall that the Pythagorean identity

\[ \sin^2 x + \cos^2 x = 1 \]

can be used to evaluate integrals of the form \( \int \sin^m x \cos^n x \, dx \) as long as either \( m \) or \( n \) is odd. Practice this technique with the following integral:

1. \( \int \sin^5 x \cos^2 x \, dx \)

Now we will develop a similar strategy for integrals of the form \( \int \tan^m x \sec^n x \, dx \). What do you get when you divide both sides of the Pythagorean identity by \( \cos^2 x \)? Simplify your answer and write the new identity here:

(\text{it should be in terms of } \tan x \text{ and } \sec x).

Now try the following integrals. Work with your group to develop a strategy for using the new identity (also called a Pythagorean identity since it comes directly from the other one) to solve these: \(^1\)

2. \( \int \tan^6 x \sec^2 x \, dx \)

over for more fun!

\(^1\)Hint. You may want to recall the derivatives of \( \tan x \) and of \( \sec x \) before you begin.

\[(\tan x)' = \text{______________________} \quad (\sec x)' = \text{______________________}\]
Write your new identity again here, for reference:

3. \[ \int \tan^2 x \sec^6 x \, dx \]

4. \[ \int \tan^3 x \sec x \, dx \]

Can you think of situations where your strategy will not work for integrals like \[ \int \tan^m x \sec^n x \, dx \]?