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Statement of Research Interests

Although my research is in the area of low-dimensional topology, in many ways I consider myself an algebraist. The techniques I use — group presentations, chain complexes, homotopy theory — are very algebraic in nature.

I became interested in the homotopy classification of two-dimensional CW complexes (or *2-complexes*, for short) after attending a talk given by Dr. Micheal Dyer, who later became my advisor. He explained how every 2-complex with fundamental group  $G$  is homotopy equivalent to a certain standard 2-complex constructed from some presentation of  $G$ . Thus the entire classification question boils down to comparing presentations of groups.

The question, though, is far from simple to answer. By inspecting two group presentations, it is fairly easy to check whether the two groups are isomorphic and whether the induced standard 2-complexes have the same Euler characteristic. After that, however, the problem of developing and using homotopy invariants to distinguish the spaces (or prove that they are the same) is much more difficult.

A  $[G, 2]$ -complex is a 2-dimensional CW complex with fundamental group isomorphic to a group  $G$ . Broadly speaking, then, my research addresses the following question:

**Question.** Given a group  $G$  and  $[G, 2]$ -complexes  $X$  and  $Y$  with the same Euler characteristic, is  $X$  homotopy equivalent to  $Y$ ?

In the case that  $G$  is finite, some results are known. In 1976, W. Metzler introduced an invariant, called *bias*, to distinguish the homotopy types of spaces with finite fundamental group  $G$ . Then, in 1979, W. Browning generalized Metzler's results and classified many  $[G, 2]$ -complexes with minimal Euler characteristic, again for  $G$  finite. Since then there have been several further developments of this theory, including results of M. Dyer and M. Lustig.

For infinite groups, however, the answers are largely unknown. My research mainly concerns the classification of  $[G, 2]$ -complexes where  $G$  is an infinite group.

For  $[G, 2]$ -complexes  $X$  and  $Y$ , the general method is to construct chain maps  $f_*: C_*(\tilde{X}) \rightarrow C_*(\tilde{Y})$  on the equivariant chain complexes of the universal covers  $\tilde{X}, \tilde{Y}$  of the two spaces. Since any homotopy equivalence  $f: X \rightarrow Y$  gives rise to a chain equivalence  $f_*: C_*(\tilde{X}) \rightarrow C_*(\tilde{Y})$ , any homotopy obstruction arising from such an equivalence must necessarily vanish in the obstruction group. Thus we must choose an obstruction group for which such obstructions pass to the quotient. We also quotient by variations arising from different choices of chain maps  $C_*(\tilde{X}) \rightarrow C_*(\tilde{X})$ , to ensure uniqueness. In order to extend this method to infinite groups, I choose a suitable rank-invariant ring  $R$  and form an obstruction group from a quotient of the  $K$ -theory group  $K_1(R)$  via a chosen ring homomorphism  $\varphi: \mathbb{Z}G \rightarrow R$ .

In my dissertation I give such a generalization of Browning's Theorem to infinite groups. I also give a second homotopy invariant which applies to infinite groups and is independent of previous results. Finally, I demonstrate the homotopy equivalence of pairs of  $[G, 2]$ -complexes, where  $G$  belongs to a certain family of 2-knot groups.

Higher-dimensional knot groups and other metacyclic groups provide a rich source of examples for pairs of presentations of groups. Variations on these presentations yield  $[G, 2]$ -complexes for which the homotopy classification is presently open. In addition, the second homotopy invariant of my dissertation may be modified in order to classify certain  $[G, 3]$ -complexes; that is, three-dimensional CW-complexes with fundamental group isomorphic to the group  $G$ . This invariant is thus quite generalizable, and could be used in a variety of contexts to determine the homotopy types of various low-dimensional spaces. In the future I hope to classify more 2- and 3-complexes using these methods and to continue to develop more useful invariants.