History of Mathematics-M143
Hmk.6/Quiz 3 Study Guide

I. (In class Quiz study)

a. Use the method of 'diminshing breadths' so find: $\sqrt[3]{2000376}$

1st Estimate: $100^3 < 2000376 < 200^3$

therefore: $x = 100 + 10b + c$

$(100+10b)^3 = 1\cdot100^3 + 3\cdot100^2\cdot10b + 3\cdot100(10\cdot b)^2 + (10b)^3 \le 2000376$

Now subtract 100^3 from both sides.

to get: $3\cdot100^2\cdot10b + 3\cdot100(10\cdot b)^2 + (10b)^3 \le 1000376$

$b=1 \rightarrow 331000$

$b=2 \rightarrow 728,000$

$b=3 \rightarrow 1197000$ too much!

so $b=2 \xrightarrow{\text{gives new}}$ estimate $\underline{120}$

$$\begin{array}{r} 1000376 \\ -\ 728000 \\ \hline 272,376 \end{array}$$

$3\cdot(120)^2\cdot c + 3\cdot120\cdot c^2 + c^3 \le 272,376$

$c=1 \rightarrow 43561$ $\quad c=2 \rightarrow 87,848 \ \ldots \ c=6 \rightarrow 272,376$ $=$ $\therefore$ Answer is $\boxed{126}$

b. (Homework challenge) Use the method of 'diminishing breadths' to find: $\sqrt[4]{279,841}$. (problem #23; Katz; p.228)

1st. Estimate

$20^4 < 279,841 < 30^4$

therefore: $x = 20 + c$

$(20+c)^4 = 20^4 + 4\cdot20^3\cdot c + 6\cdot20^2c^2 + 4\cdot20c^3 + c^4 \le 279,841$

Now subtract 20^4 from both sides.

$-160,000$

$$\begin{array}{r} 279,841 \\ -160,000 \\ \hline \end{array}$$

$4\cdot20^3c + 6\cdot20^2c^2 + 4\cdot20c^3 + c^4 \le 119,841$

$c=1 \rightarrow 34481$ $\quad c=2 \rightarrow 74,256$ $\quad c=3 \rightarrow 119841$

$\therefore$ Answer: $\boxed{23.}$

II. Use the method of 'Excess and Deficit' to solve the following problems:

a. Providing your own guesses, solve: $3x + 2 = 56$.

1st guess

My guesses Are : $x = 20 = a_1$

$3 \cdot 20 + 2 = 62$
$- 56$
$6 = c_1$
'excess'

2nd guess

$x = 10 = a_2$
$3 \cdot 10 + 2 = 32$

deficient. $-24 = c_2$

$$\therefore x = \frac{a_2 c_1 - a_1 c_2}{c_1 - c_2} = \frac{10(6) - 20(-24)}{6 - (-24)}$$

$$= \frac{60 + 480}{30} = \frac{540}{30} = \boxed{18}$$

b. Several people purchased a common item. If each paid 11 coins the excess is 10. If each paid 6 coins the deficiency is 15. How many people were there and what is the price of an item?

$P \cdot C = I$

people, coins, item price.

like

$\underline{\underline{a}} \cdot x = c$

$a_1 = 11$
$c_1 = 10$
$a_2 = 6$
$c_2 = -15$

solve for a using $a = \frac{c_1 - c_2}{a_1 - a_2}$

$a = \frac{10 - (-15)}{11 - 6} = \frac{25}{5} = 5.$

$\therefore$ There Are $\boxed{\text{5 people.}}$

$5 \cdot 11 = 55$ is 'excess' 10

so item price is $\boxed{45}$