

Mathematics 143/Hmk.4(SOLUTIONS)

1. From Katz, p. 191; 7, 8.

#7. Let  $x = \text{Diophantus' age at death}$ ; then  $x = \frac{1}{6}x + \frac{1}{12}x + \frac{1}{7}x + 5 + \frac{1}{2}x + 4$  it follows that  $9x = 756$  and so  $x = 84$ .

#8. To solve  $x + y = 20$ ;  $xy = 96$ , we set  $x = 10 + z$ ,  $y = 10 - z$ . Then  $100 - z^2 = 96$ ;  $z^2 = 4$ ; and  $z = 2$ . Thus,  $x = 12$  and  $y = 8$ .

2. From Book II, problem 22 of Arithmetica: Find two numbers such that the square of either added to the sum of both gives a square. [Hint: If the numbers are taken to be  $x$  and  $x + 1$ , then one condition is satisfied].

Solution: If  $x^2 + 4x + 2 = (x - 2)^2$  then the numbers are  $\frac{1}{4}$  and  $\frac{5}{4}$ .

3. Which of the following diophantine equations can not be solved?

- a.  $6x + 51y = 22$       No;  $3 \nmid 22$
- b.  $33x + 14y = 115$     Yes;  $1 \mid 115$
- c.  $14x + 35y = 93$       No;  $7 \nmid 93$

4. Determine all solutions in integers of the following diophantine equations:

- a.  $56x + 72y = 40$

Solution:  $x = 20 + 9t$ ;  $y = -15 - 7t$ .

- b.  $24x + 138y = 18$

Solution:  $x = 18 + 23t$ ;  $y = -3 - 4t$ .