Math 143-QUIZ 5
Burger/CSU, Fresno

Solution Key

1. Find the **second order** Taylor approximation of $\sqrt[3]{63}$ using the base point $a = 64$. Round answer to 4 decimal places.

$f^{(0)}(x) = x^{1/3} \xrightarrow{a=64} 64^{1/3} = \boxed{4}$

$f^{(1)}(x) = \frac{1}{3(x^{1/3})^2} \xrightarrow{a=64} \frac{1}{3 \cdot 4^2} = \boxed{\frac{1}{48}}$

$f^{(2)}(x) = \left(\frac{1}{3}x^{-2/3}\right)' = -\frac{2}{9}x^{-5/3} \xrightarrow{a=64} \boxed{\frac{-1}{4608}}$

$= -\frac{2}{9}x^{-5/3}$

$\sqrt[3]{63} \approx 4 + \frac{1}{48}\frac{(63-64)^1}{1} - \frac{1}{4608}\cdot\frac{(1)^2}{2} \approx \boxed{3.9751}$

2. Approximate the given integral using the following fourth-order McClaurin expansion (round to 4 decimal places).

$cosx \approx 1 - \frac{x^2}{2} + \frac{x^4}{24}$. $\quad \int_0^1 \cos(x^3)\,dx$

$\cos(x^3) \approx 1 - \frac{(x^3)^2}{2} + \frac{(x^3)^4}{24} = 1 - \frac{x^6}{2} + \frac{x^{12}}{24}$

$\int_0^1 \cos(x^3)dx \approx \int_0^1 1\,dx - \frac{1}{2}\int_0^1 x^6\,dx + \frac{1}{24}\int_0^1 x^{12}\,dx$

$\approx x\Big|_0^1 - \frac{1}{2}\frac{x^7}{7}\Big|_0^1 + \frac{1}{24}\frac{x^{13}}{13}\Big|_0^1 = 1 - \frac{1}{14} + \frac{1}{312}$

$\approx \boxed{0.9318}$

3. Use Fermat's method to find the derivative of $f(x) = x^3 + 3x - 1$.

$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

$f'(x) = \lim_{h\to 0}\frac{(x+h)^3+3(x+h)-1-[x^3+3x-1]}{h}$

$= \lim_{h\to 0}\frac{x^3+3x^2h+3xh^2+h^3+3x+3h-1-x^3+3x+1}{h}$

$= \lim_{h\to 0} 3x^2 + 3xh + h^2 + 3 = \boxed{3x^2+3}$

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Solutions key

4.

+	0	1	2	3	4	5	6
0	0	1	2	3	4	5	6
1	1	2	3	4	5	6	0
2	2	3	4	5	6	0	1
3	3	4	5	6	0	1	2
4	4	5	6	0	1	2	3
5	5	6	0	1	2	3	4
6	6	0	1	2	3	4	5

*	0	1	2	3	4	5	6
0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6
2	0	2	4	6	1	3	5
3	0	3	6	2	5	1	4
4	0	4	1	5	2	6	3
5	0	5	3	1	6	4	2
6	0	6	5	4	3	2	1

Complete the following tables for the finite field $\mathbb{Z}_7$ and then solve the following equations using only the defined operations above (show your steps).

i) $3x + 2 = 4$

$+5 \quad +5$

$3x = 2$

$5 \cdot 3x = 2 \cdot 5$

$x = 3$

ii) $4x + 6 = 1$

$+1 \quad +1$

$4x = 2$

$2 \cdot \quad \cdot 2$

$x = 4$

Mathematics 143/Final Exam study guide/Sp'09

1. Solvability and solutions to Diophantine equations.
2. Squaring, with compass and straightedge, of polygonal figures.
3. Row reduction; solving for coefficients of polynomials using a matrix.
4. Descartes' Rule of Signs
5. Finding roots of cubics using **depression** technique.
6. Approximation with Newton's Method.
7. Approximation of integrals and limits with Taylor/Mc Laurin Series
8. Using Fermat's definition to find derivative.
9. Algebraic structures: working in integers mod p. When a field?