

# MATH 141

## MIDTERM EXAM II

November 6, 2001

NAME (please print legibly): \_\_\_\_\_

Your University ID Number: \_\_\_\_\_

Circle your Instructor's Name along with the Lecture Time:

Felea (2 o'clock)   Knightly (9 o'clock)   Kojcinovic (10 o'clock)   Voloshina (2 o'clock)

- No calculators are allowed on this exam.
- Please show all your work. You may use back pages if necessary. You may not receive full credit for a correct answer if there is no work shown.
- Please put your simplified final answers in the spaces provided.

QUESTION	VALUE	SCORE
1	10	
2	10	
3	9	
4	9	
5	16	
6	16	
7	10	
8	10	
9	5	
10	5	
TOTAL	100	

1. (10 pts) Find the tangent line to the graph of  $f(x) = \frac{e^x}{x^2}$  at  $(1, e)$ .

ANSWER: \_\_\_\_\_

2. (10 pts) Let  $f(x) = x^2 + 2x$ .

(a) Find  $f'(x)$  using the definition of a derivative.

ANSWER: \_\_\_\_\_

(b) Find  $x$  at which the tangent line to the graph of  $f(x)$  is horizontal.

ANSWER: \_\_\_\_\_

3. (9 pts) Compute the following limits.

(a)  $\lim_{x \rightarrow 0} \sin(x + \sin x)$

ANSWER: \_\_\_\_\_

(b)  $\lim_{x \rightarrow \pi} \sqrt{2 + \cos x}$

ANSWER: \_\_\_\_\_

(c)  $\lim_{x \rightarrow \infty} \frac{x^3 + 2x - 9}{6x^3 - 6x + 4}$

ANSWER: \_\_\_\_\_

4. (9 pts) Compute the following limits.

(a)  $\lim_{x \rightarrow \infty} \frac{2x}{\sqrt[4]{x^4 + 3}}$

ANSWER: \_\_\_\_\_

(b)  $\lim_{x \rightarrow -\infty} \frac{3x^3 + 2x}{x^2 + 1}$

ANSWER: \_\_\_\_\_

(c)  $\lim_{x \rightarrow -\infty} \left( x + \sqrt{x^2 + 2x} \right)$

ANSWER: \_\_\_\_\_

5. (16 pts) Differentiate the following functions.

(a)  $\left(\sin \frac{\pi}{6}\right)^3$

ANSWER: \_\_\_\_\_

(b)  $e^x \left(x^2 + \frac{1}{\sqrt{x}}\right)$

ANSWER: \_\_\_\_\_

(c)  $\frac{x^3 + 17}{1 + \frac{1}{x}}$

ANSWER: \_\_\_\_\_

(d)  $\frac{x^2 + \sin x}{\sqrt{x}}$

ANSWER: \_\_\_\_\_

**6. (16 pts)** After kicking a ball up a steep hill John waits for it to roll back down. Assuming that its distance from John is given by  $f(t) = -2t^2 + 8t$ , answer the following questions.

(a) What is the velocity of the ball at time  $t$ ?

ANSWER: \_\_\_\_\_

(b) At what time is the velocity of the ball zero?

ANSWER: \_\_\_\_\_

(c) How far from John is the ball at a moment when it turns around?

ANSWER: \_\_\_\_\_

(d) At what time does the ball come back to John?

ANSWER: \_\_\_\_\_

7. (10 pts) Assuming that the amount of bacteria at time  $t$  is given by  $f(t) = 2e^t + 3t^2 + 10$ , find the rate of growth of the bacterial colony when  $t = 10$ .

ANSWER: \_\_\_\_\_

8. (10 pts) Find  $c$  with which

$$f(x) = \begin{cases} x^2 - c^2 & \text{if } x \leq 2 \\ 4x - 7 & \text{if } x > 2 \end{cases}$$

is a continuous function. With  $c$  you find, determine whether  $f$  is differentiable at  $x = 2$ ?

ANSWER: \_\_\_\_\_

9. (5 pts) Use the Intermediate Value Theorem to show that the equation  $x^4 - x - 1 = 0$  has a root in the closed interval  $[1, 2]$ .

10. (5 pts) Find the horizontal and the vertical asymptotes of  $f(x) = \frac{x^2 + 1}{x^2 - 1}$ .

ANSWER: \_\_\_\_\_