MATH 141

MIDTERM EXAM II ANSWER KEY

November 7, 2000

1. **(20pts)** Suppose that $f(x) = x^3 - 2x + 1$.

   (a) What is $f'(x)$?

   $$\frac{d}{dx}(x^3 - 2x + 1) = 3x^2 - 2$$

   ANSWER: $3x^2 - 2$

   (b) At what places is the tangent line to the graph of $f(x)$ horizontal ?

   $$f'(x) = 0 \implies 3x^2 - 2 = 0 \implies 3x^2 = 2 \implies x^2 = \frac{2}{3}$$

   ANSWER: $\sqrt{\frac{2}{3}}$, $-\sqrt{\frac{2}{3}}$

   (c) At what places is the slope of the tangent line equal to 1 ?

   $$f'(x) = 1 \implies 3x^2 - 2 = 1 \implies 3x^2 = 3 \implies x^2 = 1$$

   ANSWER: $+1$, -1

   (d) Find the tangent line to the graph when $x = 2$.
   The slope is

   $$f'(2) = 3(2)^2 - 2 = 12 - 2 = 10,$$

   and the point is $(2, f(2)) = (2, 5)$.

   ANSWER: $y - 5 = 10(x - 2)$

2. **(10pts)** Answer each of the following questions.

(a) Let $f(x) = \dfrac{x-2}{x+2}$. What is $f'(0)$?

$$f'(x) = \frac{(x+2)-(x-2)}{(x+2)^2} = \frac{4}{(x+2)^2} \implies f'(0) = \frac{4}{2^2} = 1$$

ANSWER: 1

(b) If $s(t) = \tan(t)$, what is $s'(\frac{\pi}{4})$?

$$s'(t) = \sec^2(t) \implies s'(\frac{\pi}{4}) = \sec^2(\frac{\pi}{4}) = \frac{1}{\cos^2(\frac{\pi}{4})} = \frac{1}{\left(\frac{\sqrt{2}}{2}\right)^2} = 2$$

ANSWER: 2

3. **(20pts)** Differentiate each of the following functions:

(a) $(x+8)(x-8)$

$$\frac{d}{dx}((x+8)(x-8)) = \frac{d}{dx}(x^2-64) = 2x$$

ANSWER: $2x$

(b) $e^x\,(1+x)$

$$\frac{d}{dx}(e^x(1+x)) = e^x(1+x) + e^x = e^x(x+2)$$

ANSWER: $e^x(1+x) + e^x$

(c) $\dfrac{\sin x}{x^2+1}$

$$\frac{d}{dx}\frac{\sin x}{x^2+1} = \frac{\cos x(x^2+1) - \sin x(2x)}{(x^2+1)^2}$$

ANSWER: $\dfrac{\cos x(x^2+1) - \sin x(2x)}{(x^2+1)^2}$

(d) $\dfrac{1}{1+\sqrt{x}}$

$$\frac{d}{dx}\frac{1}{1+\sqrt{x}} = \frac{-\frac{1}{2\sqrt{x}}}{(1+\sqrt{x})^2}$$

ANSWER: $\dfrac{-1}{2\sqrt{x}(1+\sqrt{x})^2}$

4. **(10pts)** Evaluate the following limits (note: some of them may be $+\infty$, $-\infty$, or may not even exist):

(a) $\lim_{h\to 0} \frac{\sin(\pi+h)-\sin(\pi)}{h}$

$$\lim_{h\to 0} \frac{\sin(\pi+h)-\sin(\pi)}{h} = (\sin)'(\pi) = \cos(\pi) = -1$$

ANSWER: -1

(b) $\lim_{x\to 0} \frac{\sin(x^3)}{x^2}$

$$\lim_{x\to 0} \frac{\sin(x^3)}{x^2} = \lim_{x\to 0} x\frac{\sin(x^3)}{x^3} = \lim_{x\to 0} x \lim_{x\to 0} \frac{\sin(x^3)}{x^3} = 0 * 1 = 0$$

ANSWER: 0

5. **(10pts)** Differentiate each of the following functions:

(a) $e^{(e^x)}$

$$\frac{d}{dx}e^{(e^x)} = e^{(e^x)}\frac{d}{dx}e^x = e^{(e^x)}e^x$$

ANSWER: $e^{(e^x)}e^x$

(b) $(\sin(x^2))^2$

$$\frac{d}{dx}(\sin(x^2))^2 = 2\sin(x^2)\cos(x^2)(2x)$$

ANSWER: $4x\sin(x^2)\cos(x^2)$

6. **(15pts)** The position of the weight attached to the end of the spring is given by $s(t) = 2\sin(2t)$.

(a) What is the velocity of the weight at time t ?

$$v(t) = s'(t) = 2\cos(2t) * 2 = 4\cos(2t)$$

ANSWER: $4\cos(2t)$

(b) At which times is the weight momentarily at rest (i.e. at which times is the velocity zero) ?

$$v(t) = 0 \implies \cos(2t) = 0 \implies 2t = \frac{\pi}{2} + k\pi \implies t = \frac{\pi}{4} + k\frac{\pi}{2}$$

where k is any integer.

ANSWER: $\frac{\pi}{4} + k\frac{\pi}{2}$ for $k \in \mathbb{Z}$

(c) What is the acceleration of the weight at time t ?

$$a(t) = v'(t) = 4(-\sin(2t) * 2) = -8\sin(2t)$$

ANSWER: $-8\sin(2t)$

7. **(15pts)** Suppose you have the following information about the functions f and g:

$f(1) = -2$	$g(1) = 3$
$f'(1) = 3$	$g'(1) = 2$
$f(3) = 2$	$g(3) = -1$
$f'(3) = -1$	$g'(3) = -1$

Use this information to find:

(a) $(f+g)'(1)$

$$(f+g)'(1) = f'(1) + g'(1) = 3 + 2$$

ANSWER: 5

(b) $(fg)'(3)$

$$(fg)'(3) = f'(3)g(3) + f(3)g'(3) = (-1)*(-1) + (2)*(-1) = 1 - 2$$

ANSWER: -1

(c) $(f \circ g)'(1)$

$$(f \circ g)'(1) = f'(g(1))g'(1) = f'(3)g'(1) = (-1)*2$$

ANSWER: -2

8. **(10pts)** Find the second derivatives of the following functions:

(a) $x \sin x$

$$f(x) = x\sin x \implies f'(x) = \sin x + x\cos x \implies f''(x) = \cos x + \cos x + x(-\sin x)$$

ANSWER: $2\cos x - x\sin x$

(b) 3^x

$$f(x) = 3^x \implies f'(x) = 3^x(\ln 3) \implies f''(x) = (3^x(\ln 3))(\ln 3)$$

NOTE: It is important to use a natural logarithm here.

ANSWER: $3^x(\ln 3)^2$