MATH 141

EXAM II WITH ANSWERS

November 18, 1999

No calculators allowed on this exam. Please show all your work.

I. In questions 1–5, circle the appropriate letter.

1. (5 pts) Let $f(x) = (x+1)^2(x+2)$. Find $f'(0)$.

$$f'(x) = 2(x+1)(x+2) + (x+1)^2$$

ANSWER: 5

2. (5 pts) Let $f(x) = \dfrac{x^3}{(x+2)^2}$. Find $f'(-1)$.

$$f'(x) = \frac{3x^2(x+2)^2 - x^3 2(x+2)}{(x+2)^4}$$

ANSWER: 5

3. (5 pts) Find the slope of the tangent line to the curve $2x^3 + 2y^3 - 9xy = 0$ at the point $(1, 2)$.

$$6x^2 + 6y^2y' - 9y - 9xy' = 0 \implies y' = \frac{-6x^2 + 9y}{6y^2 - 9x}$$

ANSWER: $\frac{4}{5}$

4. (5 pts) Let $f(x) = x \sin x$. Find $f''(0)$.

$$f'(x) = \sin x + x \cos x \implies f''(x) = \cos x + \cos x + x(-\sin x) = 2\cos x - x \sin x$$

ANSWER: 2

5. (5 pts) If $f(x) = \ell n((x^2+1)^5)$, then $f'(2) = ?$.

$$f'(x) = \frac{1}{(x^2+1)^5} 5(x^2+1)^4(2x) = \frac{10x}{x^2+1}$$

ANSWER: 4

II. Find the derivatives of the following functions.

6. (5 pts) $f(x) = x \ln\ x$ $\quad f'(x) = \ln\ x + 1$

7. (5 pts) $f(x) = \dfrac{x^2+1}{x}$ $\quad f'(x) = 1 - \dfrac{1}{x^2}$

8. (5 pts) $f(x) = \cos(x^3)$ $\quad f'(x) = -3\,x^2\ \sin(x^3)$

9. (5 pts) $f(x) = \cos^3(x)$ $\quad f'(x) = -3\cos(x)^2\ \sin(x)$

III. Find the derivatives of the following functions.

10. (5 pts) $f(x) = 3^x$ $\quad f'(x) = 3^x\ \log(3)$

11. (5 pts) $f(x) = x^{\sin x}$ $\quad f'(x) = x^{\sin(x)}\left(\cos(x)\ \log(x) + \dfrac{\sin(x)}{x}\right)$

12. (5 pts) $f(x) = \ln(\sin(e^x))$ $\quad f'(x) = e^x\ \cot(e^x)$

13. (5 pts) $f(x) = \sin^{-1}(2x)$ (eg. $\arcsin(2x)$) $\quad f'(x) = \dfrac{2}{\sqrt{1-4\,x^2}}$

14. (12 pts) A ball is thrown straight up with an initial velocity of 48 ft/sec from the top of a building 160 ft high. Its height above the ground $s(t)$, at time t (in seconds) is given by

$$s(t) = -16t^2 + 48t + 160 \ .$$

Note that:

$$v(t) = 48 - 32\,t$$

(a) What is the maximum height of the ball?

The maximum is achieved when $t = 3/2$. So the maximum height is $s(3/2) = 196$.

ANSWER: 196

(b) What is the velocity of the ball when it hits the ground?

The ball hits the ground for t for which

$$s(t) = -16t^2 + 48t + 160 = 0$$

.

This happens when $t = -2$ or $t = 5$. So, the answer is $v(5) = -112$.

ANSWER: -112

15. (11 pts) A cylindrical water tank has a radius of 12 inches. If water flows into the tank at a rate of 2 cubic inches per second, how fast is the water level in the tank rising?

Let $h(t)$ be a function describing the water level height. Let the area be denoted by A. Then, $A = 12^2\pi = 144\pi$. The volume is then $V(t) = Ah(t)$. The rate is constant so

$$\frac{V(t_2) - V(t_1)}{t_2 - t_1} = 2.$$

Thus,

$$\frac{Ah(t_2) - Ah(t_1)}{t_2 - t_1} = 2.$$

Dividing by A gives:

$$\frac{h(t_2) - h(t_1)}{t_2 - t_1} = \frac{2}{A} = \frac{2}{144\pi}.$$

ANSWER: $\frac{1}{72\pi}$

16. (12 pts) A light is on the ground 40 meters from a building. A man 2 meters tall walks from the light toward the building at 3/2 meters/sec.

(a) Find the height of his shadow as a function of the elapsed time since he leaves the light.

The man's position is given by $s(t) = \frac{3}{2}t$. Let $h(t)$ represent the height of the shadow.

By similarity of the triangles involved we have that

$$\frac{h(t)}{40} = \frac{2}{s(t)}.$$

Thus,

$$h(t) = \frac{80}{s(t)} = \frac{80}{\frac{3}{2}t}.$$

ANSWER: $\frac{160}{3t}$

(b) Find the rate of change of the length of his shadow when he is 20 meters from the building.

Note that he is 20 meters away when $t = 40/3$.

Then,

$$h'(t) = \frac{d}{dt}\frac{160}{3t} = -\frac{160}{3t^2} \implies h'(40/3) = -\frac{3}{10}.$$

ANSWER: $-\frac{3}{10}$