

# MATH 141

## EXAM I

October 12, 1999

No calculators allowed on this exam. Please show all your work.

1. (10 pts total) Fill in the blanks:

$$. \quad \circ = \frac{3\pi}{2}$$

$$135^\circ = \quad \text{radians}$$

$$60^\circ = \quad \text{radians}$$

$$\tan \pi =$$

$$\sin \pi/3 =$$

2. (8 pts) Let  $h(x) = \ln(x^2+1)$ . Find functions  $f(x)$  and  $g(x)$  such that  $h(x) = (f \circ g)(x)$ , i.e.  $h(x) = f(g(x))$ .

$$f(x) = \underline{\hspace{10em}}$$

$$g(x) = \underline{\hspace{10em}}$$

3. (9 pts)

(a) Find the slope  $m$  of the line through the two points  $(1, 4)$  and  $(3, 10)$ .

$$\text{ANSWER: } \underline{\hspace{10em}}$$

(b) Write the equation of this line.

$$\text{ANSWER: } \underline{\hspace{10em}}$$

(c) What is the equation of the line perpendicular to the line in parts (a) and (b) that goes through the origin.

$$\text{ANSWER: } \underline{\hspace{10em}}$$

4. (8 pts) Consider the one-to-one function  $f(x) = x^2 + 1$  with domain  $[0, \infty)$ .

(a) Find  $f^{-1}(x)$ .

ANSWER: \_\_\_\_\_

(b) State the domain of  $f^{-1}(x)$ .

ANSWER: \_\_\_\_\_

5. (8 pts) Find all  $x$  such that

(a)  $3^x = 2^{x^2}$

ANSWER: \_\_\_\_\_

(b)  $\ln(\ln x) = 2$

ANSWER: \_\_\_\_\_

6. (9 pts) Find the exact value of the following:

(a)  $\log_5 10 + \log_5 20 - 3 \log_5 2$

ANSWER: \_\_\_\_\_

(b)  $e^{\ln 5 + \ln 3}$

ANSWER: \_\_\_\_\_

(c)  $\ln e^{3.1}$

ANSWER: \_\_\_\_\_

7. (9 pts) The position of a ball at time  $t$ , measured in seconds, is given by the formula

$$s(t) = t^2 + 3t + 1$$

measured in feet.

(a) What is the average velocity of the ball between the times  $t = 1$  and  $t = 3$ ?

ANSWER: \_\_\_\_\_

(b) What is the average velocity between the times  $t = 1$  and  $t = 1 + h$ ? Simplify your answer as much as possible.

ANSWER: \_\_\_\_\_

(c) What is the instantaneous velocity of the ball at time  $t = 1$ ?

ANSWER: \_\_\_\_\_

8. (10 pts) Let  $f(x)$  be the function whose graph is shown:

(a) Find the following limits if they exist. If a limit does not exist, indicate this.

(i)  $\lim_{x \rightarrow 5} f(x)$

(ii)  $\lim_{x \rightarrow 2} f(x)$

(iii)  $\lim_{x \rightarrow 1^+} f(x)$

(iv)  $\lim_{x \rightarrow 1^-} f(x)$

(v)  $\lim_{x \rightarrow 0^+} f(x)$

(vi)  $\lim_{x \rightarrow -2^-} f(x)$

(b) At which value(s) of  $x$  is  $f(x)$  not continuous?

ANSWER: \_\_\_\_\_

9. (10 pts) Evaluate the following limits. Write DNE if the limit does not exist.

(a)  $\lim_{x \rightarrow 1} \frac{x^2 - 3x + 2}{x^2 + 2x - 3}$

ANSWER: \_\_\_\_\_

(b)  $\lim_{x \rightarrow 1} \frac{x^2 + 6x - 7}{x^2 - 3x - 4}$

ANSWER: \_\_\_\_\_

(c)  $\lim_{x \rightarrow 4} \frac{4 - x}{2 - \sqrt{x}}$

ANSWER: \_\_\_\_\_

(d)  $\lim_{x \rightarrow 3} \frac{x^2 + 6x + 8}{x^2 - 2x - 3}$

ANSWER: \_\_\_\_\_

(e)  $\lim_{x \rightarrow 2^-} \frac{x - 2}{|x - 2|}$

ANSWER: \_\_\_\_\_

10. (9 pts) Consider the function  $f(x) = \frac{2x^3}{(x + 2)^2(x - 1)}$ .

(a) Find the equation(s) of all vertical asymptotes to the graph of  $f(x)$ .

ANSWER: \_\_\_\_\_

(b) Find the equation(s) of all horizontal asymptotes to the graph of  $f(x)$ .

ANSWER: \_\_\_\_\_

**11. (10 pts)**

(a) State the limit definition of the derivative of a function  $f(x)$  at a point  $a$ , i.e.  $f'(a)$ .

ANSWER: \_\_\_\_\_

(b) Using the definition in part (a), calculate

$$f'(2) \text{ if } f(x) = \frac{1}{x+1} .$$

(Note: you must use the definition.)

ANSWER: \_\_\_\_\_