MATH 141

EXAM I

October 12, 1999

No calculators allowed on this exam. Please show all your work.

1. (10 pts total) Fill in the blanks:

\[ \frac{\pi}{2} = \frac{3\pi}{2} \]

135° = \text{ radians}

60° = \text{ radians}

\[ \tan \pi = \]

\[ \sin \frac{\pi}{3} = \]

2. (8 pts) Let \( h(x) = \ln(x^2 + 1) \). Find functions \( f(x) \) and \( g(x) \) such that \( h(x) = (f \circ g)(x) \), i.e. \( h(x) = f(g(x)) \).

\[ f(x) = \]

\[ g(x) = \]

3. (9 pts)

(a) Find the slope \( m \) of the line through the two points (1, 4) and (3, 10).

ANSWER: ____________________________

(b) Write the equation of this line.

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(c) What is the equation of the line perpendicular to the line in parts (a) and (b) that goes through the origin.

ANSWER: ____________________________
4. (8 pts) Consider the one-to-one function \( f(x) = x^2 + 1 \) with domain \([0, \infty)\).
   
   (a) Find \( f^{-1}(x) \).
   
   ANSWER: ____________________________
   
   (b) State the domain of \( f^{-1}(x) \).
   
   ANSWER: ____________________________

5. (8 pts) Find all \( x \) such that
   
   (a) \( 3^x = 2x^2 \)
   
   ANSWER: ____________________________
   
   (b) \( \ln(\ln x) = 2 \)
   
   ANSWER: ____________________________

6. (9 pts) Find the exact value of the following:
   
   (a) \( \log_5 10 + \log_5 20 - 3 \log_5 2 \)
   
   ANSWER: ____________________________
   
   (b) \( e^{\ln 5 + \ln 3} \)
   
   ANSWER: ____________________________
   
   (c) \( \ln e^{3.1} \)
   
   ANSWER: ____________________________

7. (9 pts) The position of a ball at time \( t \), measured in seconds, is given by the formula

   \[ s(t) = t^2 + 3t + 1 \]

   measured in feet.

   (a) What is the average velocity of the ball between the times \( t = 1 \) and \( t = 3 \)?

   ANSWER: ____________________________

   (b) What is the average velocity between the times \( t = 1 \) and \( t = 1 + h \)? Simplify your answer as much as possible.

   ANSWER: ____________________________

   (c) What is the instantaneous velocity of the ball at time \( t = 1 \)?

   ANSWER: ____________________________
8. (10 pts) Let $f(x)$ be the function whose graph is shown:

(a) Find the following limits if they exist. If a limit does not exist, indicate this.

(i) $\lim_{x \to 5} f(x)$

(ii) $\lim_{x \to 2} f(x)$

(iii) $\lim_{x \to 1^+} f(x)$

(iv) $\lim_{x \to 1^-} f(x)$

(v) $\lim_{x \to 0^+} f(x)$

(vi) $\lim_{x \to -2^-} f(x)$

(b) At which value(s) of $x$ is $f(x)$ not continuous?

ANSWER: _______________________

9. (10 pts) Evaluate the following limits. Write DNE if the limit does not exist.

(a) $\lim_{x \to 1} \frac{x^2 - 3x + 2}{x^2 + 2x - 3}$

ANSWER: _______________________

(b) $\lim_{x \to 1} \frac{x^2 + 6x - 7}{x^2 - 3x - 4}$

ANSWER: _______________________

(c) $\lim_{x \to 4} \frac{4 - x}{2 - \sqrt{x}}$

ANSWER: _______________________

(d) $\lim_{x \to 3} \frac{x^2 + 6x + 8}{x^2 - 2x - 3}$

ANSWER: _______________________

(e) $\lim_{x \to 2^-} \frac{x - 2}{|x - 2|}$

ANSWER: _______________________

10. (9 pts) Consider the function $f(x) = \frac{2x^3}{(x + 2)^2(x - 1)}$.

(a) Find the equation(s) of all vertical asymptotes to the graph of $f(x)$.

ANSWER: _______________________

(b) Find the equation(s) of all horizontal asymptotes to the graph of $f(x)$.

ANSWER: _______________________

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11. (10 pts)

(a) State the limit definition of the derivative of a function \( f(x) \) at a point \( a \), i.e. \( f'(a) \).

\[ \text{ANSWER: } \]

(b) Using the definition in part (a), calculate

\[ f'(2) \text{ if } f(x) = \frac{1}{x+1}. \]

(Note: you must use the definition.)

\[ \text{ANSWER: } \]