MATH 141

EXAM I WITH ANSWERS

October 12, 1999

No calculators allowed on this exam. Please show all your work.

1. (10 pts total) Fill in the blanks:
   \[ 270^\circ = \frac{3\pi}{2} \]
   \[ 135^\circ = \frac{3\pi}{4} \text{ radians} \]
   \[ 60^\circ = \frac{\pi}{3} \text{ radians} \]
   \[ \tan \pi = 0 \]
   \[ \sin \frac{\pi}{3} = \frac{\sqrt{3}}{2} \]

2. (8 pts) Let \( h(x) = \ln(x^2+1) \). Find functions \( f(x) \) and \( g(x) \) such that \( h(x) = (f \circ g)(x) \), i.e. \( h(x) = f(g(x)) \).

   \[ f(x) = \ln(x) \]
   \[ g(x) = x^2 + 1 \]

3. (9 pts)
   (a) Find the slope \( m \) of the line through the two points (1,4) and (3,10).

   ANSWER: 3

   (b) Write the equation of this line.

   ANSWER: \( y = 3x + 1 \)

   (c) What is the equation of the line perpendicular to the line in parts (a) and (b) that goes through the origin.

   ANSWER: \( y = -x/3 \)
4. (8 pts) Consider the one-to-one function \( f(x) = x^2 + 1 \) with domain \([0, \infty)\).
   (a) Find \( f^{-1}(x) \).
   \[ \text{ANSWER: } \sqrt{x - 1} \]
   (b) State the domain of \( f^{-1}(x) \).
   \[ \text{ANSWER: } [1, \infty) \]

5. (8 pts) Find all \( x \) such that
   (a) \( 3^x = 2^{x^2} \)
   \[ \text{ANSWER: } x = 0, x = \ln(3)/\ln(2) \]
   (b) \( \ln(\ln x) = 2 \)
   \[ \text{ANSWER: } x = e^{e^2} \]

6. (9 pts) Find the exact value of the following:
   (a) \( \log_5 10 + \log_5 20 - 3 \log_5 2 \)
   \[ \text{ANSWER: } 2 \]
   (b) \( e^{\ln 5 + \ln 3} \)
   \[ \text{ANSWER: } 15 \]
   (c) \( \ln e^{3.1} \)
   \[ \text{ANSWER: } 3.1 \]

7. (9 pts) The position of a ball at time \( t \), measured in seconds, is given by the formula
   \[ s(t) = t^2 + 3t + 1 \]
   measured in feet.
   (a) What is the average velocity of the ball between the times \( t = 1 \) and \( t = 3 \)?
   \[ \text{ANSWER: } 3 \]
   (b) What is the average velocity between the times \( t = 1 \) and \( t = 1 + h \)? Simplify your answer as much as possible.
   \[ \text{ANSWER: } 5 + h \]
   (c) What is the instantaneous velocity of the ball at time \( t = 1 \)?
   \[ \text{ANSWER: } 5 \]
8. (10 pts) Let \( f(x) \) be the function whose graph is shown:

[GRAPH NOT AVAILABLE]

(a) Find the following limits if they exist. If a limit does not exist, indicate this.

(i) \( \lim_{x \to 5} f(x) \)

(ii) \( \lim_{x \to 2} f(x) \)

(iii) \( \lim_{x \to 1^+} f(x) \)

(iv) \( \lim_{x \to 1^-} f(x) \)

(v) \( \lim_{x \to 0^+} f(x) \)

(vi) \( \lim_{x \to -2^-} f(x) \)

(b) At which value(s) of \( x \) is \( f(x) \) not continuous?

ANSWER:

9. (10 pts) Evaluate the following limits. Write DNE if the limit does not exist.

(a) \( \lim_{x \to 1} \frac{x^2 - 3x + 2}{x^2 + 2x - 3} \) ANSWER: \(-\frac{1}{4}\)

(b) \( \lim_{x \to 1} \frac{x^2 + 6x - 7}{x^2 - 3x - 4} \) ANSWER: 0

(c) \( \lim_{x \to 4} \frac{4 - x}{2 - \sqrt{x}} \) ANSWER: 4

(d) \( \lim_{x \to 3} \frac{x^2 + 6x + 8}{x^2 - 2x - 3} \) ANSWER: DNE

(e) \( \lim_{x \to 2^-} \frac{x - 2}{|x - 2|} \) ANSWER: \(-1\)
10. (9 pts) Consider the function \( f(x) = \frac{2x^3}{(x + 2)^2(x - 1)}. \)

(a) Find the equation(s) of all vertical asymptotes to the graph of \( f(x) \).

\[ \text{ANSWER: } x = 1, \ x = -2 \]

(b) Find the equation(s) of all horizontal asymptotes to the graph of \( f(x) \).

\[ \text{ANSWER: } y = 2 \]

11. (10 pts)

(a) State the limit definition of the derivative of a function \( f(x) \) at a point \( a \), i.e. \( f'(a) \).

\[ \text{ANSWER: } \lim_{h \to 0} \frac{f(a+h) - f(a)}{h} \]

(b) Using the definition in part (a), calculate

\[ f'(2) \text{ if } f(x) = \frac{1}{x + 1}. \]

(Note: you must use the definition.)

\[ \text{ANSWER: } 1/9 \]