MATH 141

EXAM I WITH ANSWERS

October 12, 1999

No calculators allowed on this exam. Please show all your work.

1. (10 pts total) Fill in the blanks:

$\underline{270^\circ} = \dfrac{3\pi}{2}$

$135^\circ = \underline{3\pi/4}$ radians

$60^\circ = \underline{\pi/3}$ radians

$\tan\pi = \underline{0}$

$\sin\pi/3 = \underline{\sqrt{3}/2}$

2. (8 pts) Let $h(x) = \ln(x^2+1)$. Find functions $f(x)$ and $g(x)$ such that $h(x) = (f \circ g)(x)$, i.e. $h(x) = f(g(x))$.

$$f(x) = \underline{\ln(x)}$$
$$g(x) = \underline{x^2+1}$$

3. (9 pts)

(a) Find the slope m of the line through the two points $(1, 4)$ and $(3, 10)$.

ANSWER: $\underline{3}$

(b) Write the equation of this line.

ANSWER: $\underline{y = 3x + 1}$

(c) What is the equation of the line perpendicular to the line in parts (a) and (b) that goes through the origin.

ANSWER: $\underline{y = -x/3}$

4. (8 pts) Consider the one-to-one function $f(x) = x^2 + 1$ with domain $[0, \infty)$.

(a) Find $f^{-1}(x)$.

ANSWER: $\underline{\sqrt{x-1}}$

(b) State the domain of $f^{-1}(x)$.

ANSWER: $\underline{[1, \infty)}$

5. (8 pts) Find all x such that

(a) $3^x = 2^{x^2}$

ANSWER: $\underline{x = 0,\ x = \ln(3)/\ln(2)}$

(b) $\ln(\ln x) = 2$

ANSWER: $\underline{x = e^{(e^2)}}$

6. (9 pts) Find the exact value of the following:

(a) $\log_5 10 + \log_5 20 - 3\log_5 2$

ANSWER: $\underline{2}$

(b) $e^{\ln 5 + \ln 3}$

ANSWER: $\underline{15}$

(c) $\ln e^{3.1}$

ANSWER: $\underline{3.1}$

7. (9 pts) The position of a ball at time t, measured in seconds, is given by the formula

$$s(t) = t^2 + 3t + 1$$

measured in feet.

(a) What is the average velocity of the ball between the times $t = 1$ and $t = 3$?

ANSWER: $\underline{3}$

(b) What is the average velocity between the times $t = 1$ and $t = 1 + h$? Simplify your answer as much as possible.

ANSWER: $\underline{5 + h}$

(c) What is the instantaneous velocity of the ball at time $t = 1$?

ANSWER: $\underline{5}$

8. (10 pts) Let $f(x)$ be the function whose graph is shown:

[GRAPH NOT AVAILABLE]

(a) Find the following limits if they exist. If a limit does not exist, indicate this.

(i) $\lim_{x\to 5} f(x)$

(ii) $\lim_{x\to 2} f(x)$

(iii) $\lim_{x\to 1^+} f(x)$

(iv) $\lim_{x\to 1^-} f(x)$

(v) $\lim_{x\to 0^+} f(x)$

(vi) $\lim_{x\to -2^-} f(x)$

(b) At which value(s) of x is $f(x)$ not continuous?

ANSWER:

9. (10 pts) Evalute the following limits. Write DNE if the limit does not exist.

(a) $\lim_{x\to 1} \dfrac{x^2-3x+2}{x^2+2x-3}$

ANSWER: $-1/4$

(b) $\lim_{x\to 1} \dfrac{x^2+6x-7}{x^2-3x-4}$

ANSWER: 0

(c) $\lim_{x\to 4} \dfrac{4-x}{2-\sqrt{x}}$

ANSWER: 4

(d) $\lim_{x\to 3} \dfrac{x^2+6x+8}{x^2-2x-3}$

ANSWER: DNE

(e) $\lim_{x\to 2^-} \dfrac{x-2}{|x-2|}$

ANSWER: -1

10. (9 pts) Consider the function $f(x) = \dfrac{2x^3}{(x+2)^2(x-1)}$.

(a) Find the equation(s) of all vertical asymptotes to the graph of $f(x)$.

ANSWER: $\underline{x = 1,\ x = -2}$

(b) Find the equation(s) of all horizontal asysmptotes to the graph of $f(x)$.

ANSWER: $\underline{y = 2}$

11. (10 pts)

(a) State the limit definition of the derivative of a function $f(x)$ at a point a, i.e. $f'(a)$.

ANSWER: $\underline{\lim_{h\to 0} \frac{f(a+h)-f(a)}{h}}$

(b) Using the definition in part (a), calculate

$$f'(2) \text{ if } f(x) = \frac{1}{x+1}\ .$$

(Note: you must use the definition.)

ANSWER: $\underline{1/9}$