MATH 141

FINAL EXAM

December 15, 1999

No calculators allowed on this exam. Please show all your work.

PART I (Part I is worth 100 points in total.)

1. (10 pts)

(a) What is the slope of the line through the two points $(2, 3)$ and $(4, 7)$?

ANSWER: ______________________

(b) What is the equation of the line which is parallel to the line in part (a) and which passes through the point $(1, 2)$?

ANSWER: ______________________

(c) What is the equation of the line which is perpendicular to the line in part (a) and which passes through the point $(1, 2)$?

ANSWER: ______________________

In questions 2–5, circle the correct answer.

2. (5 pts) Let $f(x) = \dfrac{4x^2 - 3}{x^2 + 2}$. Find $f'(x)$.

(a) $\dfrac{8x(x^2+2) + (4x^2-3)2x}{(x^2+2)^2}$

(b) $\dfrac{8x(x^2+2) - (4x^2-3)2x}{(x^2+2)^2}$

(c) $\dfrac{(4x^2-3)2x - 8x(x^2+2)}{(x^2+2)^2}$

(d) $-(x^2+2)8x - 2x(4x^2-3)$

(e) none of these

3. (5 pts) Let $g(x) = (5x^3 - 2)^{-1/3}$. Find $g'(x)$.

(a) $-\frac{1}{3}\,(5x^3 - 2)^{-4/3}$ (b) $\frac{45}{2}\,(5x^3 - 2)^{2/3}x^2$

(c) $-5x^2(5x^3 - 2)^{-4/3}$ (d) $\frac{1}{10x^2}\,(5x^3 - 2)^{2/3}$

(d) none of these

4. (5 pts) Let $g(x)' = (1 + \tan x)^{3/2}$. Find $g'(x)$.

(a) $3(1 + \tan x)^{1/2}\cot x$ (b) $\frac{3}{2}\,(1 + \tan x)^{1/2}$

(c) $\frac{3}{2}\,(1 + \tan x)^{1/2}\sec^2 x$ (d) $\frac{3}{2}\,(1 + \sec^2 x)^{1/2}$

(d) none of these

5. (5 pts) Let $f(x) = \sin^{-1}(e^x)$. Find $f'(x)$.

(a) $\frac{e^x}{\sqrt{1 - x^2}}$ (b) $\frac{x}{\sqrt{1 - x^2}}$

(c) $\frac{e^x}{\sqrt{1 - e^{2x}}}$ (d) $\frac{x}{\sqrt{1 + x^2}}$ (d) none of these

Problems 6 and 7 refer to the function $h(x) = \dfrac{4x^2 - 2x}{3 - 3x^2}$ **.**

In these two questions, there may be more than one correct answer, so circle all answers which apply.

6. (5 pts) The horizontal asymptotes of $h(x)$ are $y =$

(a) 1 (b) $\frac{1}{2}$ (c) 0 (d) -1 (e) $\frac{-4}{3}$

7. (5 pts) The vertical asymptotes of $h(x)$ are $x =$

(a) 1 (b) $\frac{1}{2}$ (c) 0 (d) -1 (e) $\frac{-4}{3}$

In questions 8–12, circle the correct answer.

8. (5 pts) Find an equation for the tangent line to the curve

$$y = x + \sin 2x \text{ at } x = 0 \, .$$

(a) $y = 2x$ (b) $y = 3x$ (c) $y = 3$

(d) $y = -2x + 1$ (e) none of these

9. (5 pts) Let the function f be defined by

$$f(x) = \begin{cases} \sqrt{4 - x^2} & \text{for } -2 < x < 2 \\ |x + 2| & \text{for all other } x \, . \end{cases}$$

Find all values of x for which f is discontinuous.

(a) -2 (b) 2 (c) for -2 and 2

(d) $\{x | x \geq 2 \text{ or } x \leq -2\}$ (e) none of these

10. (5 pts) Find y' if y is defined implicitly by the equation $y + \sin y = \cos(x^2)$.

(a) $y' = \dfrac{-\sin(x^2)}{1 + \cos y}$ (b) $y' = \dfrac{-2x\sin(x^2)}{1 + \cos y}$

(c) $y' = 2x\sin(x^2)$ (d) $y' = -\left(\dfrac{1 + 2x\sin(x^2)}{\cos y}\right)$

(e) $y' = 1 + \cos y + 2x\sin(x^2)$

A 7-foot-tall man walks towards a lamppost at the rate of 3 feet per second. The lamp is 21 feet above the ground. Let x be the man's distance from the lamppost, and let s be the length of his shadow.

11. (5 pts) Find a formula for s in terms of x.

(a) $s = \dfrac{1}{3}\,x$ (b) $s = \dfrac{1}{2}\,x$ (c) $s = \sqrt{49 + x^2}$

(d) $s = \dfrac{3}{x}$ (e) none of these

12. (5 pts) How fast is the length of his shadow changing?

(a) 3 feet per second (b) -1 foot per second

(c) -3 feet per second (d) $-\frac{3}{2}$ feet per second (e) none of these

In questions 13–15, circle the correct answer.

13. (5 pts) Let $f(x) = \sin(x^2 - 1)$. Find $f'(1)$.

(a) 0 (b) 1 (c) 2 (d) 3 (e) 4 (f) 5

14. (5 pts) Let $f(x) = \cos(x)\sin(x)$. Find $f'(0)$.

(a) 0 (b) 1 (c) 2 (d) 3 (e) 4 (f) 5

15. (5 pts) Find the slope of the tangent line to the curve $x\cos y + y\sin(x-1) + 1 = 0$ at the point $\left(1, \frac{\pi}{2}\right)$.

(a) 0 (b) 1 (c) -1 (d) $\pi/2$ (e) -2 (f) π

16. (10 pts) Evaluate the following limits. Write DNE if the limit does not exist.

(a) $\lim_{x\to 4} \frac{x^2 - 5x + 4}{x^2 - 6x + 8}$

ANSWER: ____________________

(b) $\lim_{x\to 2} \frac{x^2 - 5x + 4}{x^2 - 6x + 8}$

ANSWER: ____________________

(c) $\lim_{x\to 0} \frac{\sin^2(3x)}{2x^2}$

ANSWER: ____________________

(d) $\lim_{x\to 0^-} \frac{|x|}{x}$

ANSWER: ____________________

(e) $\lim_{x\to 2} \frac{\sqrt{x} - \sqrt{2}}{x - 2}$

ANSWER: ____________________

17. (10 pts) A rock is thrown up from a cliff 64 ft. high with a velocity of 48 ft./sec. Its' height above the ground (in feet) at time t (in seconds) is given by

$$s(t) = -16t^2 + 48t + 64$$

(a) Find the velocity of the rock at time t.

ANSWER: ______________________

(b) What is the maximum height the rock reaches?

ANSWER: ______________________

(c) Find the velocity of the rock when it is 32 ft. above the ground.

ANSWER: ______________________

(d) At what time does the rock hit the ground?

ANSWER: ______________________

END OF PART I

PART II (Part II is worth 100 points in total.)

18. (10 pts) Which of the following statements are true? Circle **T** or **F** as appropriate.

T F $\ell n(e^x) = x$ for all real x

T F $\sqrt{x^2} = x$ for all real x

T F $\ell n(e^{x^2}) = 2x$ for all real x

T F $e^{\ell n(x)+\ell n(y)} = xy$ if $x > 0$ and $y > 0$

19. (12 pts) Water is pouring into an inverted conical tank at a constant rate of 2 ft^3 per minute. If the tank is 10 ft. deep and has a radius of 5 ft. at the top, how fast (in ft. per minute) is the water level in the tank rising when the water is 6 ft. deep? Hint: the volume of a cone of height h and radius of the base r is $\frac{1}{3}\pi r^2 h$. Circle the correct answer.

ANSWERS:

(a) $\frac{1}{10\pi}$ (b) $\frac{2}{9\pi}$ (c) $\frac{1}{10}$ (d) $\frac{1}{2\pi}$

(e) $\frac{1}{5}$ (f) $\frac{1}{2}$ (g) 1 (h) $\frac{\pi}{2}$

20. (12 pts) Suppose $f(10) = 4$ and $f'(10) = 60$. Use linear approximations to estimate $f(10.2)$.

ANSWER: ______________________

21. (10 pts) Which of the following statements are TRUE and which are FALSE? Circle **T** or **F** as appropriate.

T F Continuous functions are always differentiable.

T F Differentiable functions are always continuous.

T F If a function has a local maximum at c, then $f'(c)$ exists and $= 0$.

T F The tangent line to a graph at a point $(c, f(c))$ never intersects the graph in more than one point.

T F If $f'(c) = 0$, then $f(c)$ must be either a local maximum or a local minimum.

22. (12 pts) Find the absolute maximum and minimum values of the function $f(x) = x^3 - 2x^2 + x$ on the interval $0 \leq x \leq 2$ and all places where they occur.

The max value of $f(x)$ is ____________________

and it occurs at $x =$ ____________________

The min value of $f(x)$ is ____________________

and it occurs at $x =$ ____________________

23. (12 pts) Which of the following are TRUE and which are FALSE? Circle **T** or **F** as appropriate.

T	F	A continuous function on an interval always has a maximum and a minimum value.
T	F	If $f'(c) = 0$ and $f''(c) > 0$, the $f(x)$ has a local maximum at c.
T	F	If $f(x)$ and $g(x)$ are two functions which are differentiable on an interval I and $f'(x) = g'(x)$ for all x in I, then $f(x) = g(x)$ for all x in I.
T	F	Assuming that the position of a particle is given by a differentiable function $s(t)$ on a time interval, then the average velocity over that interval is equal to the instantaneous velocity at some time in that interval.

24. (32 pts) Parts (a) through (g) refer to the function $f(x) = x^4 - 8x^3$. In these eight parts, there may be more than one correct answer, so <u>circle all answers that apply</u>. In parts (d) through (g), you may need to specify more than one choice in order to combine intervals for a complete answer. (Each part is worth 4 points.)

(a) At what values of x does the graph of f have a horizontal tangent line?

(a) 0 (b) 4 (c) 6 (d) 8 (e) none of these

(b) At what values of x does $f(x)$ have <u>critical</u> points?

(a) 0 (b) 4 (c) 6 (d) 8 (e) none of these

(c) For what x does the graph of f have an inflection point?

(a) 0 (b) 4 (c) 6 (d) 8 (e) none of these

(d) For what x intervals is f increasing?

(a) $(-\infty, 0)$ (b) $(0, 4)$ (c) $(4, 6)$ (d) $(6, 8)$ (e) $(8, +\infty)$

(e) For what x intervals is f decreasing?

(a) $(-\infty, 0)$ (b) $(0, 4)$ (c) $(4, 6)$ (d) $(6, 8)$ (e) $(8, +\infty)$

(f) For what x intervals is f concave upwards?

(a) $(-\infty, 0)$ (b) $(0, 4)$ (c) $(4, 6)$ (d) $(6, 8)$ (e) $(8, +\infty)$

(g) For what x intervals is f concave downwards?

(a) $(-\infty, 0)$ (b) $(0, 4)$ (c) $(4, 6)$ (d) $(6, 8)$ (e) $(8, +\infty)$