MATH 141
Final Exam
May 7, 2001
Part I

1. (8 points)

(a) Solve the inequality \(17x + 3 < 14x - 2\)
(b) Write the equation of the line parallel to the line \(4x - 6y = 3\) that goes through \((1, 1)\).

2. (10 points) Find the exact value of:

(a) \(\tan(\pi)\)
(b) \(\sin^2(3) + \cos^2(3)\)
(c) \(\csc\left(\frac{\pi}{2}\right)\)
(d) \(\frac{1}{2}\ln 9 + \ln 5 - \ln 15\)
(e) \(2^{\log_2 3 + \log_2 7}\)

3. (10 points)

(a) Let \(f(x) = \arcsin(2x)\) and \(g(x) = x^3\). Compute \(h(x) = (f \circ g)(x)\)
(b) Find the inverse of \(h\).

4. (10 points) Solve the equations:

(a) \(\ln x - \ln x^3 = -4\)
(b) \(5^{x^2 - 4} = 125\)
5. **(8 points)** Find $c$ such that the function $f$ is continuous on $\mathbb{R}$.

\[
f(x) = \begin{cases} 
x^2 - c & , x \leq 5 \\
cx + 6 & , x > 5
\end{cases}
\]

6. **(24 points)** Compute the limits (do not use L’Hospital’s Rule):

   (a) \( \lim_{x \to 3} \frac{x^2 - 7x + 12}{x^2 - 9} \)

   (b) \( \lim_{x \to 4^+} \frac{x^2 + 3x}{(x - 4)(x + 7)} \)

   (c) \( \lim_{x \to 5} \frac{\frac{1}{x} - \frac{1}{5}}{x - 5} \)

   (d) \( \lim_{x \to \infty} \frac{6x^3 - 3x^2 + 4}{x^3 + 7x - 5} \)

   (e) \( \lim_{x \to \infty} \frac{x^7 + 10}{x^4 + 3} \)

   (f) \( \lim_{x \to 0} x^4 \sin \left( \frac{1}{2x} \right) \)

   (g) \( \lim_{x \to 0} \frac{\sin 4x}{9x} \sin 6x \)

   (h) \( \lim_{x \to 0} \frac{\cos x - 1}{e^x \tan(2x)} \)

7. **(12 points)** Let \( f(x) = \frac{x}{x - 3} \)

   (a) Find the vertical and horizontal asymptotes of the graph of \( f \).

   (b) Find the derivative of \( f \) using the definition.

   (c) Find the tangent line to the graph of \( f \) at \( (4, 4) \).

8. **(12 points)** Find the derivatives of:

   (a) \( f(x) = e^{4x} \tan 2x - \sqrt{x} \cos x^2 \)
\[(b) \quad g(x) = \frac{2 \cot x - x^6}{x^3 + 5} \]
\[(c) \quad h(x) = 4^{\cos 5x} + (\cos 5x)^4 \]
\[(d) \quad k(x) = \sqrt[3]{\sin (e^{-x})} \]

9. (6 points) The cost function of producing \( x \) units of some commodity is \( C(x) = 1000 + 23x + 0.002x^3 \). What is the marginal cost at the production level of 400 units?

**Part II**

1. (9 points) Differentiate the following functions:
   \[(a) \quad f(x) = \arcsin(2x) \]
   \[(b) \quad g(x) = e^{\arctan x} \]
   \[(c) \quad h(x) = \log_3 (-\sin x) \]

2. (12 points) Let \( f(x) = x^3 \ln x \). Find \( f'(x) \), \( f''(x) \), \( f'''(x) \), and \( f^{(4)}(x) \).

3. (10 points) If \( xy^3 - x^2y^2 + 2y = -8 \) and \( y(3) = 2 \), find \( y'(3) \).

4. (10 points) Let \( f(x) = 5(x^2 + 1)^3(\cos x)^4x \). Use logarithmic differentiation to find \( f'(x) \).

5. (10 points) Find the linearization of \( f(x) = x^{3/2} \) at \( x = 4 \) and use it to approximate \( (4.02)^{3/2} \).

6. (12 points) Car A starts moving north at 0.5 km/min from a point \( P \). At the same time car B starts moving west at 1 km/min from a point 10 km due east of \( P \). At what rate is the distance between the
cars changing 6 minutes later? Is the distance increasing or decreasing at this instant?

7. **(15 points)** Let \( f(x) = x^4 + 2x^3 - 5 \).

   (a) Find the critical numbers of \( f(x) \).

   (b) Where is \( f(x) \) increasing? Decreasing?

   (c) Find local maxima and minima of \( f(x) \).

   (d) Where is \( f(x) \) concave upward? Concave downward?

   (e) Find the inflection points of \( f(x) \).

8. **(10 points)** Show that the equation \( x^5 + 3x^3 + 5x + 7 = 0 \) has exactly one root in the interval \([-1, 1]\).

9. **(12 points)** Evaluate the following limits:

   (a) \( \lim_{x \to 0} \frac{\tan x}{e^x - 1} \)

   (b) \( \lim_{x \to \infty} x^2 e^{-2x} \)

   (c) \( \lim_{x \to 0^+} (-\ln x)^x \)