MATH 141
Final Exam
May 7, 2001
Part I

1. **(8 points)**

   (a) Solve the inequality $17x + 3 < 14x - 2$

   (b) Write the equation of the line parallel to the line $4x - 6y = 3$ that goes through $(1, 1)$.

2. **(10 points)** Find the exact value of:

   (a) $\tan(\pi)$

   (b) $\sin^2(3) + \cos^2(3)$

   (c) $\csc\left(\frac{\pi}{2}\right)$

   (d) $\frac{1}{2}\ln 9 + \ln 5 - \ln 15$

   (e) $2^{\log_2 3 + \log_2 7}$

3. **(10 points)**

   (a) Let $f(x) = \arcsin(2x)$ and $g(x) = x^3$. Compute $h(x) = (f \circ g)(x)$

   (b) Find the inverse of h.

4. **(10 points)** Solve the equations:

   (a) $\ln x - \ln x^3 = -4$

   (b) $5^{x^2-4} = 125$

5. **(8 points)** Find c such that the function f is continuous on $\mathbb{R}$.

$$f(x) = \begin{cases} x^2 - c & , x \leq 5 \\ cx + 6 & , x > 5 \end{cases}$$

6. **(24 points)** Compute the limits (do not use L'Hospital's Rule):

   (a) $\displaystyle \lim_{x \to 3} \frac{x^2 - 7x + 12}{x^2 - 9}$

   (b) $\displaystyle \lim_{x \to 4^+} \frac{x^2 + 3x}{(x-4)(x+7)}$

   (c) $\displaystyle \lim_{x \to 5} \frac{\frac{1}{x} - \frac{1}{5}}{x - 5}$

   (d) $\displaystyle \lim_{x \to \infty} \frac{6x^3 - 3x^2 + 4}{x^3 + 7x - 5}$

   (e) $\displaystyle \lim_{x \to -\infty} \frac{x^7 + 10}{x^4 + 3}$

   (f) $\displaystyle \lim_{x \to 0} x^4 \sin\left(\frac{1}{2x}\right)$

   (g) $\displaystyle \lim_{x \to 0} \frac{\sin 4x}{9 \sin 6x}$

   (h) $\displaystyle \lim_{x \to 0} \frac{\cos x - 1}{e^x \tan(2x)}$

7. **(12 points)** Let $f(x) = \dfrac{x}{x-3}$

   (a) Find the vertical and horizontal asymptotes of the graph of f.

   (b) Find the derivative of f using the definition.

   (c) Find the tangent line to the graph of f at $(4, 4)$.

8. **(12 points)** Find the derivatives of:

   (a) $f(x) = e^{4x} \tan 2x - \sqrt{x} \cos x^2$

(b) $g(x) = \dfrac{2\cot x - x^6}{x^3 + 5}$

(c) $h(x) = 4^{\cos 5x} + (\cos 5x)^4$

(d) $k(x) = \sqrt[3]{\sin\left(e^{-x}\right)}$

9. **(6 points)** The cost function of producing x units of some commodity is $C(x) = 1000 + 23x + 0.002x^3$. What is the marginal cost at the production level of 400 units?

Part II

1. **(9 points)** Differentiate the following functions:

   (a) $f(x) = \arcsin(2x)$

   (b) $g(x) = e^{\arctan x}$

   (c) $h(x) = \log_3(-\sin x)$

2. **(12 points)** Let $f(x) = x^3 \ln x$. Find $f'(x)$, $f''(x)$, $f'''(x)$, and $f^{(4)}(x)$.

3. **(10 points)** If $xy^3 - x^2y^2 + 2y = -8$ and $y(3) = 2$, find $y'(3)$.

4. **(10 points)** Let $f(x) = 5(x^2+1)^3(\cos x)^{4x}$. Use logarithmic differentiation to find $f'(x)$.

5. **(10points)** Find the linearization of $f(x) = x^{3/2}$ at $x = 4$ and use it to approximate $(4.02)^{3/2}$.

6. **(12 points)** Car A starts moving north at 0.5 km/min from a point P. At the same time car B starts moving west at 1 km/min from a point 10 km due east of P. At what rate is the distance between the

cars changing 6 minutes later? Is the distance increasing or decreasing at this instant?

7. **(15 points)** Let $f(x) = x^4 + 2x^3 - 5$.

   (a) Find the critical numbers of $f(x)$.

   (b) Where is $f(x)$ increasing? Decreasing?

   (c) Find local maxima and minima of $f(x)$.

   (d) Where is $f(x)$ concave upward? Concave downward?

   (e) Find the inflection points of $f(x)$.

8. **(10 points)** Show that the equation $x^5 + 3x^3 + 5x + 7 = 0$ has exactly one root in the interval $[-1, 1]$.

9. **(12 points)** Evaluate the following limits:

   (a) $\lim_{x \to 0} \frac{\tan x}{e^x - 1}$

   (b) $\lim_{x \to \infty} x^2 e^{-2x}$

   (c) $\lim_{x \to 0^+} (-\ln x)^x$