MATH 141
Midterm 2
april 5, 2001

1. **(20 points)** Compute the limits:

   (a) $\lim_{x\to-\infty} \dfrac{6x^2+5x}{(1-x)(2x-3)}$

   (b) $\lim_{x\to\infty} \dfrac{x^2+4x+1}{x^5-3x^3+4}$

   (c) $\lim_{x\to\infty} \sqrt{x^2+2x+2}-x$

   (d) $\lim_{x\to\infty} (x+\sqrt{x})(x^2+4)$

   (e) $\lim_{x\to-\infty} e^{-\frac{3}{x^2}}$

2. **(10 points)** Find the horizontal asymptotes of the graph of the function $f(x)=\frac{x-3}{\sqrt{x^2+3x+2}}$

3. **(10points)** Let $f(x)=x^2-\frac{2}{x}$.

   (a) Find $f'(x)$ using the definition of the derivative.

   (b) Find the tangent to the graph of f at $(1,-1)$.

   (c) At what point of the graph is the tangent horizontal?

4. **(12 points)** Compute the derivatives of

   (a) $f(x)=e^x\cos x+x^2\sqrt[3]{x}-7\tan x+\frac{1}{x}$

   (b) $g(x)=\frac{\sqrt{x}-2x+4}{x^3+12}$

   (c) $h(x)=\frac{2\sin x}{\tan x-4\cos x}$

5. **(8 points)** A particle moves along a straight line and its position at time t is $s(t)=t^3-9t^2+15t+10$

   (a) Find the velocity of the particle at time $t=2$.

   (b) When is the particle at rest?

6. **(6 points)** The volume of a cube with side s is $V(s) = s^3$. What is the rate of change of the volume with respect to s when $s = 5$?

7. **(12 points)** Compute the limits:

   (a) $\lim_{x \to 0} \dfrac{\cot 3x}{\csc x}$

   (b) $\lim_{x \to 0} \dfrac{\sin^2 x}{2x}$

   (c) $\lim_{x \to 0} \dfrac{\cos x - 1}{x^2 + 4x}$

8. **(10 points)** Where is the function f differentiable?

$$f(x) = \begin{cases} x + 4 & , x \leq 2 \\ x^2 - 2x + 6 & , x > 2 \end{cases}$$

9. **(12 points)** Find the derivatives of:

   (a) $f(x) = \sqrt{\tan x + 2x}$

   (b) $g(x) = \sin^2(\cos x)$

   (c) $h(x) = 2^{x^2 + 3 \sin x}$

   (d) $k(x) = \cos\left(e^{\frac{1}{x}}\right)$