MATH 141

MIDTERM EXAM II

April 3rd, 2000

- **No calculators are allowed on this exam.**
- **Please show all your work. You may not receive full credit for a correct answer if there is no work shown.**
- **Please put your final answer in the boxes when provided**

1. **(13pts)** Use the **definition of the derivative** (i.e. as the limit of difference quotients) to find the derivative of $f(x) = (3x+1)^2$ at $x = 1$, that is $f'(1)$.

$$f'(1) = \lim_{h \to 0} \frac{f(1+h) - f(1)}{h} = \lim_{h \to 0} \frac{24\,h + 9\,h^2}{h} = \lim_{h \to 0} 24 + 9\,h = 24$$

2. **(12pts)**

(a) If $y = \dfrac{3x^2+1}{\sqrt{x}}$, find $\dfrac{dy}{dx}$.

ANSWER: $\dfrac{-1 + 9\,x^2}{2\,x^{\frac{3}{2}}}$

(b) If $s = \dfrac{t^2}{t+3}$, find $\dfrac{ds}{dt}$.

ANSWER: $\dfrac{t\,(6+t)}{(3+t)^2}$

(c) If $y = 3^x$, find $y'(x)$.

ANSWER: $3^x \, \log(3)$

3. **(9pts)** Suppose you have the following information about the functions f and g:

$f(2) = 2$	$g(2) = 8$
$f'(2) = 3$	$g'(2) = -1$
$f(6) = 2$	$g(6) = 2$
$f'(6) = -\frac{1}{2}$	$g'(6) = -4$

Use this information to find:

(a) $(fg)'(6)$

$$(fg)'(6) = (f'g+fg')(6) = f'(6)g(6)+f(6)g'(6) = -1/2*2+2*(-4) = -1-8 = -9$$

(b) $(f-g)'(2)$

$$(f-g)'(2) = f'(2) - g'(2) = 3 - (-1) = 4$$

(c) $(f \circ g)'(6)$

$$(f \circ g)'(6) = f'(g(6))g'(6) = f'(2)(-4) = 3 * (-4) = -12$$

4. **(16pts)** Suppose the position of a car along a certain road is given by

$$s(t) = \frac{2}{3}t^3 - 6t^2 + 16t$$

where time is measured in seconds.

(a) Find $v(t)$, the velocity of the car as a function of t.

$$v(t) = s'(t) = 16 - 12\,t + 2\,t^2$$

(b) Find $a(t)$, the acceleration of the car as a function of t.

$$a(t) = v'(t) = -12 + 4\,t$$

(c) When is the first time the car is at rest?

$$v(t) = 0 \iff 16 - 12\,t + 2\,t^2 = 0$$

Therefore, $t = 2$ or $t = 4$. The first time is when $t = 2$.

(d) After 3.5 seconds, is the car speeding up or slowing down? Justify your answer.

For t bigger than 3, the acceleration is positive and the velocity is increasing. Nevertheless, because the velocity is negative at 3.5, it is actually decreasing in magnitude, so the car is slowing down (recall that the speed is the magnitude of the velocity).

5. **(20pts)** For each of the following, find $\frac{dy}{dx}$:

(a) $y = x^2 \sin x$

$$\frac{dy}{dx} = x^2\ \cos(x) + 2\,x\ \sin(x)$$

(b) $\frac{\sin x}{\cos x}$

$$\frac{dy}{dx} = \sec(x)^2$$

(c) $y = e^{\sqrt{2}}$

$$\frac{dy}{dx} = 0$$

(d) $y = \cos(xe^x)$

$$\frac{dy}{dx} = -\left(e^x + e^x\,x\right)\ \sin(e^x\,x)$$

6. **(5pts)** Find the limit:

$$\lim_{x\to 0} \frac{2\sin(3x)}{x}$$

$$\lim_{x\to 0} \frac{2\sin(3x)}{x} = 6\lim_{x\to 0} \frac{\sin(3x)}{3x} = 6\lim_{z\to 0} \frac{\sin(z)}{z} = 6$$

7. **(10pts)** What is the equation of the tangent line to the curve $y = \frac{1}{1+x^2}$ at the point on the curve when $x = 1$?

$$\frac{dy}{dx} = \frac{-2\,x}{\left(1+x^2\right)^2} \implies \frac{dy}{dx}(1) = -\frac{1}{2}$$

The line goes through the point $(1, y(1)) = (1, \frac{1}{2})$, so

$$y - \frac{1}{2} = -\frac{1}{2}(x-1)$$

8. **(15pts)**

(a) Find the second derivative of $y = (x^2+2)^3$ at $x = 0$.

$$\begin{aligned} y &= (x^2+2)^3 \implies \\ \frac{dy}{dx} &= 6\,x\,\left(2+x^2\right)^2 \implies \\ \frac{d^2y}{dx^2} &= 24\,x^2\ \left(2+x^2\right) + 6\left(2+x^2\right)^2 \implies \\ y''(0) &= 24 \end{aligned}$$

(b) If $\sin y = x$, find $\frac{dy}{dx}$.
Well, $\sin y = x$ implies that $y = \arcsin x$, so

$$\frac{dy}{dx} = \frac{d}{dx}\arcsin x = \frac{1}{\sqrt{1-x^2}}$$