

# MATH 141

## MIDTERM EXAM I

February 21, 2000

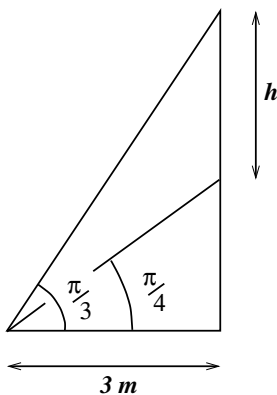
- No calculators are allowed on this exam.
- Please show all your work. You may not receive full credit for a correct answer if there is no work shown.
- Please put your final answer in the boxes provided

1. (8pts) Let  $P$  be the point  $(1, 2)$  and  $Q$  be the point  $(-3, 4)$ .

(a) What is the slope of the line  $L$  joining  $P$  to  $Q$ ?

(b) The midpoint between  $P$  and  $Q$  is  $(-1, 3)$ . Find the equation of the line which is perpendicular to  $L$  and which goes through this midpoint.

2. (8pts) Find the value of  $h$  in the diagram below:



3. (15pts)

(a) Solve the following equation for  $x$ :

$$\ln x + \ln(x - 2) = \ln(2x)$$

(b) If  $t$  is the **number of years since 1990**, a population of rabbits is given by the equation  $p(t) = 100e^{kt}$  for some constant  $k$ , and the population doubles every  $\frac{1}{4}$  of a year.

i. What was the population in 1990?

ii. Find the constant  $k$ .

4. (20pts) Evaluate the following limits (note: some of them may be  $+\infty$  or  $-\infty$ ).

(a)  $\lim_{x \rightarrow 2} \frac{x^2 + 5x + 6}{x + 2}$

(b)  $\lim_{x \rightarrow 3} \frac{\frac{1}{3} - \frac{1}{x}}{x - 3}$

(c)  $\lim_{x \rightarrow 1^-} \frac{2x^2 + x + 1}{(x - 1)(x + 2)}$

(d)  $\lim_{x \rightarrow 1^+} \frac{2x^2 + x + 1}{(x - 1)(x + 2)}$

(e)  $\lim_{x \rightarrow -3^+} \frac{3x^2 + 7}{(x + 1)(x + 3)}$

5. (8pts) Find the horizontal asymptote of the function  $y = \frac{4x^2 - 3}{8x^2 - 6x}$ . Justify how you obtain your answer by showing your work.

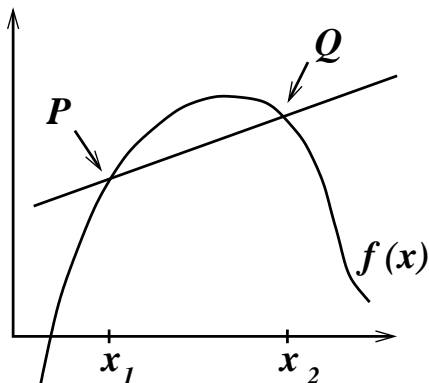
6. (13pts)

(a) Find  $\lim_{x \rightarrow 3^+} \frac{|3 - x|}{2x - 6}$ .

(b) What value of  $c$  will make the following function continuous at  $x = 2$ ?

$$f(x) = \begin{cases} \frac{2x^2 - 2x - 12}{x + 2} & \text{if } x > 2 \\ cx + c & \text{if } x \leq 2 \end{cases}$$

7. (5pts) In the picture below, give the formula for the slope of the secant line which goes through the points  $P$  and  $Q$  on the graph of  $y = f(x)$ .

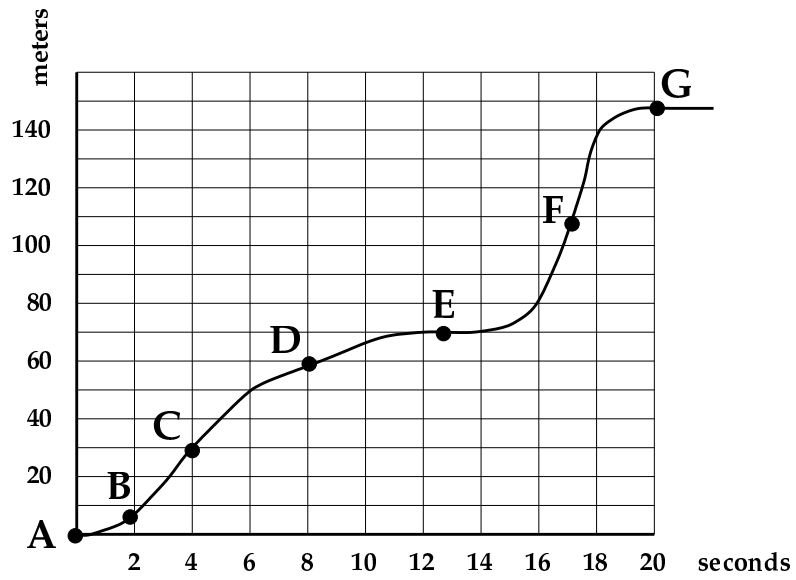


8. (15pts) A rock is dropped from the roof of a building, and its height in meters is given by  $s(t) = -5t^2 + 30$  where  $t$  is measured in seconds.

(a) What is the average velocity of the rock during the interval from  $t = 1$  to  $t = 2$  seconds?

(b) Use the **definition of instantaneous rate of change** to find the velocity of the rock at  $t = 1$  second,  $v(1)$ .

9. (8pts) In the diagram below, the position of a car is given as a function of time (time is given in seconds and distance in meters). Use the diagram to answer the following questions, giving letters or pairs of letters as answers:



(a) When is the car travelling the fastest.

(b) Give an interval when the car is speeding up (e.g. "between J and K").

(c) Give two letters at which times the car is stationary.

(d) Estimate the velocity of the car at C.