

MATH 141

MIDTERM EXAM I

February 21, 2000

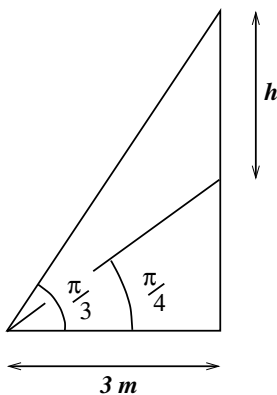
- No calculators are allowed on this exam.
- Please show all your work. You may not receive full credit for a correct answer if there is no work shown.
- Please put your final answer in the boxes provided

1. (8pts) Let P be the point $(1, 2)$ and Q be the point $(-3, 4)$.

(a) What is the slope of the line L joining P to Q ?

(b) The midpoint between P and Q is $(-1, 3)$. Find the equation of the line which is perpendicular to L and which goes through this midpoint.

2. (8pts) Find the value of h in the diagram below:



3. (15pts)

(a) Solve the following equation for x :

$$\ln x + \ln(x - 2) = \ln(2x)$$

(b) If t is the **number of years since 1990**, a population of rabbits is given by the equation $p(t) = 100e^{kt}$ for some constant k , and the population doubles every $\frac{1}{4}$ of a year.

i. What was the population in 1990?

ii. Find the constant k .

4. (20pts) Evaluate the following limits (note: some of them may be $+\infty$ or $-\infty$).

(a) $\lim_{x \rightarrow 2} \frac{x^2 + 5x + 6}{x + 2}$

(b) $\lim_{x \rightarrow 3} \frac{\frac{1}{3} - \frac{1}{x}}{x - 3}$

(c) $\lim_{x \rightarrow 1^-} \frac{2x^2 + x + 1}{(x - 1)(x + 2)}$

(d) $\lim_{x \rightarrow 1^+} \frac{2x^2 + x + 1}{(x - 1)(x + 2)}$

(e) $\lim_{x \rightarrow -3^+} \frac{3x^2 + 7}{(x + 1)(x + 3)}$

5. (8pts) Find the horizontal asymptote of the function $y = \frac{4x^2 - 3}{8x^2 - 6x}$. Justify how you obtain your answer by showing your work.

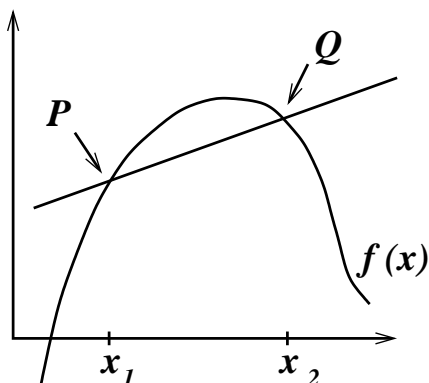
6. (13pts)

(a) Find $\lim_{x \rightarrow 3^+} \frac{|3 - x|}{2x - 6}$.

(b) What value of c will make the following function continuous at $x = 2$?

$$f(x) = \begin{cases} \frac{2x^2 - 2x - 12}{x + 2} & \text{if } x > 2 \\ cx + c & \text{if } x \leq 2 \end{cases}$$

7. (5pts) In the picture below, give the formula for the slope of the secant line which goes through the points P and Q on the graph of $y = f(x)$.

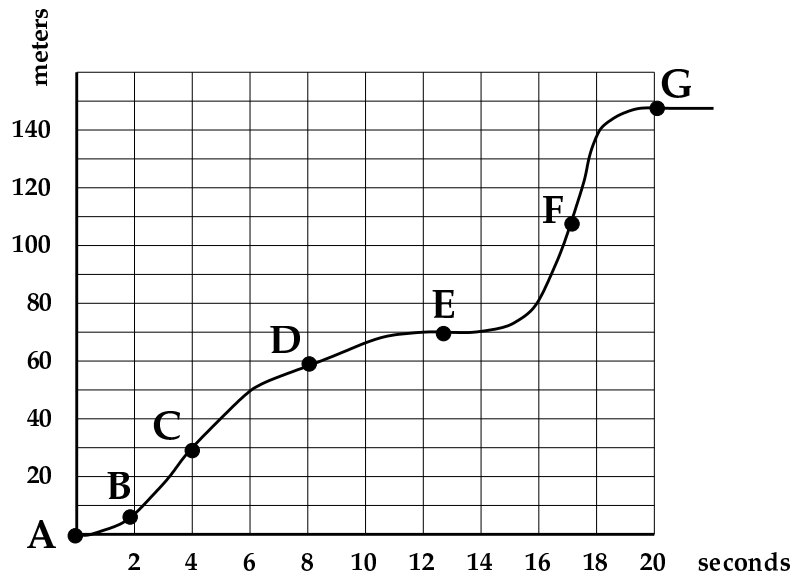


8. (15pts) A rock is dropped from the roof of a building, and its height in meters is given by $s(t) = -5t^2 + 30$ where t is measured in seconds.

(a) What is the average velocity of the rock during the interval from $t = 1$ to $t = 2$ seconds?

(b) Use the **definition of instantaneous rate of change** to find the velocity of the rock at $t = 1$ second, $v(1)$.

9. (8pts) In the diagram below, the position of a car is given as a function of time (time is given in seconds and distance in meters). Use the diagram to answer the following questions, giving letters or pairs of letters as answers:



(a) When is the car travelling the fastest.

(b) Give an interval when the car is speeding up (e.g. "between J and K").

(c) Give two letters at which times the car is stationary.

(d) Estimate the velocity of the car at C.