MATH 141

MIDTERM EXAM I WITH ANSWERS

February 21, 2000

• No calculators are allowed on this exam.
• Please show all your work. You may not receive full credit for a correct answer if there is no work shown.
• Please put your final answer in the boxes provided

1. **(8pts)** Let \( P \) be the point \((1, 2)\) and \( Q \) be the point \((-3, 4)\).

   (a) What is the slope of the line \( L \) joining \( P \) to \( Q? \)

   \[-1/2\]

   (b) The midpoint between \( P \) and \( Q \) is \((-1, 3)\). Find the equation of the line which is perpendicular to \( L \) and which goes through this midpoint.

   \[y = 2x + 5\]

2. **(8pts)** Find the value of \( h \) in the diagram below:

   \[3(\sqrt{3} - 1)\]
3. (15pts)

(a) Solve the following equation for \( x \):
\[
\ln x + \ln(x - 2) = \ln(2x)
\]

\( x = 4 \)

(b) If \( t \) is the number of years since 1990, a population of rabbits is given by the equation \( p(t) = 100e^{kt} \) for some constant \( k \), and the population doubles every \( \frac{1}{4} \) of a year.

i. What was the population in 1990?

\( 100 \)

ii. Find the constant \( k \).

\( 4 \ln 2 \)

4. (20pts) Evaluate the following limits (note: some of them may be \( +\infty \) or \( -\infty \)).

(a) \( \lim_{x \to 2} \frac{x^2 + 5x + 6}{x + 2} \)

\( 5 \)

(b) \( \lim_{x \to 3} \frac{\frac{1}{3} - \frac{1}{x}}{x - 3} \)

\( \frac{1}{9} \)

(c) \( \lim_{x \to 1^-} \frac{2x^2 + x + 1}{(x - 1)(x + 2)} \)

\( -\infty \)

(d) \( \lim_{x \to 1^+} \frac{2x^2 + x + 1}{(x - 1)(x + 2)} \)

\( \infty \)

(e) \( \lim_{x \to -3^+} \frac{3x^2 + 7}{(x + 1)(x + 3)} \)

\( -\infty \)

5. (8pts) Find the horizontal asymptote of the function \( y = \frac{4x^2 - 3}{8x^2 - 6x} \). Justify how you obtain your answer by showing your work.

\( y = 1/2 \)
6. (13pts)

(a) Find \( \lim_{x \to 3} \frac{|3 - x|}{2x - 6} \).

(b) What value of \( c \) will make the following function continuous at \( x = 2 \)?

\[
f(x) = \begin{cases} 
2x^2 - 2x - 12 & \text{if } x > 2 \\
x + 2 & \text{if } x \leq 2 \\

\end{cases}
\]

\[c = -2/3\]

7. (5pts) In the picture below, give the formula for the slope of the secant line which goes through the points \( P \) and \( Q \) on the graph of \( y = f(x) \).

\[
\frac{f(x_2) - f(x_1)}{x_2 - x_1}
\]

8. (15pts) A rock is dropped from the roof of a building, and its height in meters is given by \( s(t) = -5t^2 + 30 \) where \( t \) is measured in seconds.

(a) What is the average velocity of the rock during the interval from \( t = 1 \) to \( t = 2 \) seconds?

\[-15\]

(b) Use the definition of instantaneous rate of change to find the velocity of the rock at \( t = 1 \) second, \( v(1) \).

\[-10\]
9. (8pts) In the diagram below, the position of a car is given as a function of time (time is given in seconds and distance in meters). Use the diagram to answer the following questions, giving letters or pairs of letters as answers:

(a) When is the car travelling the fastest.  
   
   \[ \text{F} \]

(b) Give an interval when the car is speeding up (e.g. “between J and K”).

   \[ \text{between E and F} \]

(c) Give two letters at which times the car is stationary.

   \[ \text{E and G} \]

(d) Estimate the velocity of the car at C.

   \[ 10 \text{m/sec} \]