MATH 141

MIDTERM EXAM I WITH ANSWERS

February 21, 2000

- **No calculators are allowed on this exam.**
- **Please show all your work. You may not receive full credit for a correct answer if there is no work shown.**
- **Please put your final answer in the boxes provided**

1. **(8pts)** Let P be the point $(1, 2)$ and Q be the point $(-3, 4)$.

   (a) What is the slope of the line L joining P to Q?

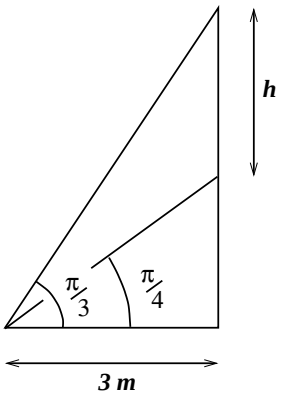
   $-1/2$

   (b) The midpoint between P and Q is $(-1, 3)$. Find the equation of the line which is perpendicular to L and which goes through this midpoint.

   $y = 2x + 5$

2. **(8pts)** Find the value of h in the diagram below:

   $3(\sqrt{3} - 1)$



3. **(15pts)**

(a) Solve the following equation for x:

$x = 4$

$$\ln x + \ln(x-2) = \ln(2x)$$

(b) If t is the **number of years since 1990**, a population of rabbits is given by the equation $p(t) = 100e^{kt}$ for some constant k, and the population doubles every $\frac{1}{4}$ of a year.

i. What was the population in 1990?

100

ii. Find the constant k.

$4 \ln 2$

4. **(20pts)** Evaluate the following limits (note: some of them may be $+\infty$ or $-\infty$).

(a) $\lim_{x \to 2} \dfrac{x^2 + 5x + 6}{x + 2}$

5

(b) $\lim_{x \to 3} \dfrac{\frac{1}{3} - \frac{1}{x}}{x - 3}$

1/9

(c) $\lim_{x \to 1^-} \dfrac{2x^2 + x + 1}{(x-1)(x+2)}$

$-\infty$

(d) $\lim_{x \to 1^+} \dfrac{2x^2 + x + 1}{(x-1)(x+2)}$

∞

(e) $\lim_{x \to -3^+} \dfrac{3x^2 + 7}{(x+1)(x+3)}$

$-\infty$

5. **(8pts)** Find the horizontal asymptote of the function $y = \dfrac{4x^2 - 3}{8x^2 - 6x}$. Justify how you obtain your answer by showing your work.

$y = 1/2$

6. **(13pts)**

(a) Find $\lim_{x \to 3^+} \dfrac{|3-x|}{2x-6}$.

$1/2$

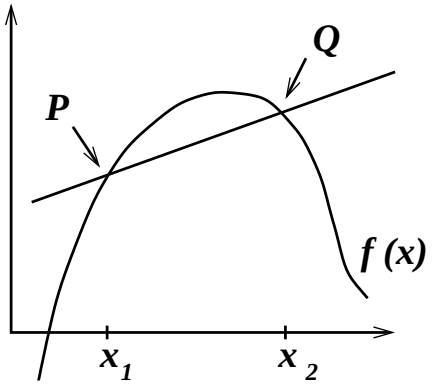
(b) What value of c will make the following function continuous at $x = 2$?

$$f(x) = \begin{cases} \dfrac{2x^2 - 2x - 12}{x+2} & \text{if } x > 2 \\ cx + c & \text{if } x \leq 2 \end{cases}$$

$c = -2/3$

7. **(5pts)** In the picture below, give the formula for the slope of the secant line which goes through the points P and Q on the graph of $y = f(x)$.

$$\frac{f(x_2) - f(x_1)}{x_2 - x_1}$$



8. **(15pts)** A rock is dropped from the roof of a building, and its height in meters is given by $s(t) = -5t^2 + 30$ where t is measured in seconds.

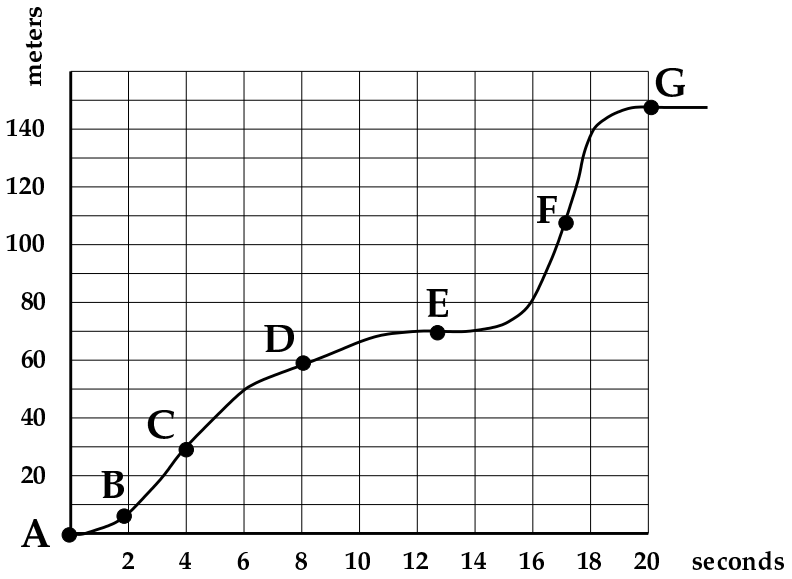
(a) What is the average velocity of the rock during the interval from $t = 1$ to $t = 2$ seconds?

-15

(b) Use the **definition of instantaneous rate of change** to find the velocity of the rock at $t = 1$ second, $v(1)$.

-10

9. **(8pts)** In the diagram below, the position of a car is given as a function of time (time is given in seconds and distance in meters). Use the diagram to answer the following questions, giving letters or pairs of letters as answers:



(a) When is the car travelling the fastest.

F

(b) Give an interval when the car is speeding up (e.g. "between J and K").

between E and F

(c) Give two letters at which times the car is stationary.

E and G

(d) Estimate the velocity of the car at **C**.

10m/sec