Math Field Day
Prep Session
Grades 9-10

Instructors: **Matt Elizondo**

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California State University, Fresno
February 21, 2019

About Us

Matt Elizondo

- Math Major, Senior
- Graduating 2020, BA Mathematics
- Goal: Teach
- Fun Fact!

Maria Nogin

- Math Faculty
- Fresno Math Circle coordinator and instructor
- Lifelong passion: math problem solving
- Have been coordinating and writing problems for Math Field Day

Definitions

Terms you should know:

- Natural Numbers
- Integers
- Rational Numbers
- Real Numbers
- Factor, Divisor, Multiple
- Divides, Divisible by

Divisibility

Divisibility tests for 2, 3, 4, 5, 8, 9, 10, 11.

Prime Factorization theorem: every positive integer larger than 1 can be written as a product of primes, uniquely up to order.

Know the prime factorization of the current year.

The number of positive divisors of $N = p_1^{k_1} p_2^{k_2} \dots p_n^{k_n}$ is $(k_1 + 1)(k_2 + 1) \dots (k_n + 1)$.

Useful formulas

(1) Difference of squares

- $a^2 - b^2 = (a - b)(a + b)$

(2) Difference/sum of cubes

- $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$
- $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

(3) Square of sum/difference

- $(a \pm b)^2 = a^2 \pm 2ab + b^2$

(4) Cube of sum/difference

- $(a \pm b)^3 = a^3 \pm 3a^2b + 3ab^2 \pm b^3$

More useful formulas

(1) Sum of Arithmetic Sequence

- $1 + 2 + 3 + \cdots + n = \dfrac{n(n+1)}{2}$

(2) Partial Sum Geometric Series

- $1 + q + q^2 + \cdots + q^n = \dfrac{1 - q^{n+1}}{1 - q}$

(3) Sum of Infinite Geometric Series

- $1 + q + q^2 + \cdots = \dfrac{1}{1 - q}$

Example 1

Compute: $1 + 2 + 3 + \cdots + 2019$.

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Solution.

$$1 + 2 + 3 + \cdots + 2019 = \frac{2019 \cdot 2020}{2} = 2019 \cdot 1010 = 2039190.$$

Example 2

If $a = 6$ and $b = 24$, find $\dfrac{a^8 - b^4}{(a^4 + b^2)(a^2 + b)}$.

Example 2

If $a = 6$ and $b = 24$, find $\dfrac{a^8 - b^4}{(a^4 + b^2)(a^2 + b)}$.

Solution.

$$\frac{a^8 - b^4}{(a^4 + b^2)(a^2 + b)} = \frac{(a^4 - b^2)(a^4 + b^2)}{(a^4 + b^2)(a^2 + b)} = \frac{(a^2 - b)(a^2 + b)(a^4 + b^2)}{(a^4 + b^2)(a^2 + b)} =$$

$$a^2 - b = 36 - 24 = 12.$$

Example 3

How many different positive factors does the number $10!$ have?

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Solution.

$10! = 1 \cdot 2 \cdot 3 \cdot 2^2 \cdot 5 \cdot (2 \cdot 3) \cdot 7 \cdot 2^3 \cdot 3^2 \cdot (2 \cdot 5) = 2^8 \cdot 3^4 \cdot 5^2 \cdot 7.$

The number of positive factors is $9 \cdot 5 \cdot 3 \cdot 2 = 270$.

Example 4

Today is Thursday. What day of the week will be exactly 2019 days from today?

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Solution.

$2019 = 288 \cdot 7 + 3$.

Answer: Sunday.

Mad Hatter

Here are the basics for this contest.

- Solo competition, rapid fire computation
- Multiple choice format, problems read aloud
- 2 minutes per problem
- 2 parts with 60 minutes each
- Pencil/paper allowed
- Correct = 1 points, blank = 0 points, incorrect = 0 points
- Highest score WINS

For this mock test, 10 questions will be given. 2 minutes per problem.

Problem 1

Determine the sum $3 + 7 + 11 + \cdots + 35$.

(a) 140
(b) 171
(c) 315
(d) 342

Problem 2

Find the first of three consecutive odd integers whose sum is 57.

(a) 13
(b) 15
(c) 17
(d) 19

Problem 3

Suppose a positive integer N is divisible by both 9 and 21. What is the smallest possible number of positive integers that divide N?

(a) 6

(b) 5

(c) 4

(d) 3

Problem 4

Which of the following numbers is a perfect square?

(a) $98!99!$

(b) $98!100!$

(c) $99!100!$

(d) $100!101!$

(e) $99!101!$

Problem 5

What is the smallest positive prime p greater than 2 such that $p^3 + 7p^2$ is a perfect square?

(a) 13
(b) 17
(c) 23
(d) 29

Problem 6

Let n be a positive integer. If n is divided by 2, 3, 4, 5, or 6, the remainder is 1, but n is divisible by 7. What is the least possible value of n?

(a) 421
(b) 721
(c) 301
(d) 63
(e) None of the above

Problem 7

Evaluate: $\dfrac{4351^2 - 4347^2}{4350 \cdot 4353 - 4351^2}$.

(a) $\frac{1}{2}$

(b) 1

(c) 2

(d) 4

(e) 8

Problem 8

Which of the following CANNOT be the sum of the digits of a square?

(a) 13
(b) 11
(c) 7
(d) 4

Problem 9

What is the tens digit of the smallest positive integer that is divisible by each of 20, 16, 2016?

(a) 0
(b) 2
(c) 4
(d) 8

Problem 10

What is the smallest positive integer $x > 100$ such that every permutation of the digits of x is prime?

(a) 101

(b) 103

(c) 113

(d) 117

Test Complete!

Take a moment, breathe, relax.

Problem 1

(MH 9-10, 2006)

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(a) 140
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(d) 342

Answer: (b)

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(MH 9-10, 2014)

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Answer: (c)

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(a) 6
(b) 5
(c) 4
(d) 3

Answer: (a)

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(MH 11-12, 2010)

Which of the following numbers is a perfect square?

(a) 98!99!
(b) 98!100!
(c) 99!100!
(d) 100!101!
(e) 99!101!

Problem 4

(MH 11-12, 2010)

Which of the following numbers is a perfect square?

(a) 98!99!
(b) 98!100!
(c) 99!100!
(d) 100!101!
(e) 99!101!

Answer: (c)

Problem 5

(MH 9-10, 2006)

What is the smallest positive prime p greater than 2 such that $p^3 + 7p^2$ is a perfect square?

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(b) 17
(c) 23
(d) 29

Problem 5

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(b) 17
(c) 23
(d) 29

Answer: (d)

Problem 6

(MH 9-10, 2017)

Let n be a positive integer. If n is divided by 2, 3, 4, 5, or 6, the remainder is 1, but n is divisible by 7. What is the least possible value of n?

(a) 421
(b) 721
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(d) 63
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(a) 421
(b) 721
(c) 301
(d) 63
(e) None of the above

Answer: (c)

Problem 7

(MH 9-10, MH 11-12 2006)

Evaluate: $\dfrac{4351^2 - 4347^2}{4350 \cdot 4353 - 4351^2}$.

(a) $\frac{1}{2}$
(b) 1
(c) 2
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(MH 9-10, MH 11-12 2006)

Evaluate: $\dfrac{4351^2 - 4347^2}{4350 \cdot 4353 - 4351^2}$.

(a) $\frac{1}{2}$
(b) 1
(c) 2
(d) 4
(e) 8

Answer: (e)

Problem 8

(MH 9-10, 2017)

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(b) 11
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(MH 9-10, 2017)

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(a) 13
(b) 11
(c) 7
(d) 4

Answer: (b)

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(MH 9-10, 2017)

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(d) 8

Answer: (d)

Problem 10

(MH 9-10, 2017)

What is the smallest positive integer $x > 100$ such that every permutation of the digits of x is prime?

(a) 101

(b) 103

(c) 113

(d) 117

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(a) 101

(b) 103

(c) 113

(d) 117

Answer: (c)

Leap Frog

- Teams of two people from the same school
- Each team member gets 10 problems
- First hour: each team member works on his/her own 10 problems, no communication is allowed
- Second hour: may communicate, exchange papers, etc.
- Turn in a single set of 20 answers from the team
- Correct $= 4$ points, blank $= 0$ points, incorrect $= -1$ points

For this mock test, each team member will get 4 problems.
20 minutes per part.

Problem 1

(LF 9-12, 2006)

The units digit of the number $9^{2006} - 3^{2006}$ is

(a) 6
(b) 4
(c) 2
(d) 0
(e) None of these

Problem 1

(LF 9-12, 2006)

The units digit of the number $9^{2006} - 3^{2006}$ is

(a) 6
(b) 4
(c) 2
(d) 0
(e) None of these

Answer: (c)

Problem 2

(LF 9-10, 2013)

How many 4-digit palindromic numbers *abba* are divisible by 9?

(a) 7
(b) 8
(c) 9
(d) 10
(e) None of these

Problem 2

(LF 9-10, 2013)

How many 4-digit palindromic numbers *abba* are divisible by 9?

(a) 7
(b) 8
(c) 9
(d) 10
(e) None of these

Answer: (d)

Problem 3

(LF 9-10, 2015)

Suppose that when dividing the number n by 7, there results a remainder of 3. What then is the remainder if you were to divide the number $2015n$ by 7?

(a) 0
(b) 1
(c) 2
(d) 3
(e) None of these

Problem 3

(LF 9-10, 2015)

Suppose that when dividing the number n by 7, there results a remainder of 3. What then is the remainder if you were to divide the number $2015n$ by 7?

(a) 0
(b) 1
(c) 2
(d) 3
(e) None of these

Answer: (e)

Problem 4

(MH 9-10, 2006)

If a, b, $a+b$, and $a-b$ are all prime numbers, which of the following statements must be true about the sum of these four numbers?

(a) The sum is odd and prime.
(b) The sum is odd and divisible by 3.
(c) The sum is odd and divisible by 7.
(d) The sum is even.

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If a, b, $a+b$, and $a-b$ are all prime numbers, which of the following statements must be true about the sum of these four numbers?

(a) The sum is odd and prime.
(b) The sum is odd and divisible by 3.
(c) The sum is odd and divisible by 7.
(d) The sum is even.

Answer: (a)

Problem 5

(LF 9-12, 2006)

Suppose n, a and b are positive integers. In order for n to divide ab, it is _____ that n divides a or n divides b.

(a) necessary and sufficient
(b) necessary, but not sufficient
(c) sufficient, but not necessary
(d) neither necessary nor sufficient
(e) None of these

Problem 5

(LF 9-12, 2006)

Suppose n, a and b are positive integers. In order for n to divide ab, it is _____ that n divides a or n divides b.

(a) necessary and sufficient
(b) necessary, but not sufficient
(c) sufficient, but not necessary
(d) neither necessary nor sufficient
(e) None of these

Answer: (c)

Problem 6

(LF 9-10, 2017)

The sum of eight consecutive integers is 212. What is the sum of the first and last integers?

(a) 52
(b) 53
(c) 54
(d) 55
(e) None of these

Problem 6

(LF 9-10, 2017)

The sum of eight consecutive integers is 212. What is the sum of the first and last integers?

(a) 52
(b) 53
(c) 54
(d) 55
(e) None of these

Answer: (b)

Problem 7

(LF 9-10, 2015)

For how many of the ten digits $x = 0, 1, 2, \ldots, 9$ is the 2017-digit number $n = 1\underbrace{xx\ldots x}_{2015}0$ divisible by 24?

(a) 0
(b) 1
(c) 2
(d) 3
(e) None of these

Problem 7

(LF 9-10, 2015)

For how many of the ten digits $x = 0, 1, 2, \ldots, 9$ is the 2017-digit number $n = 1\underbrace{xx\ldots x}_{2015}0$ divisible by 24?

(a) 0
(b) 1
(c) 2
(d) 3
(e) None of these

Answer: (b)

Problem 8

(LF 2014 9-10)

The digit sum of a number is the sum of its decimal digits. For example, the digit sum of the number 3206 is $3 + 2 + 0 + 6 = 11$. Determine the digit sum of the number $(10^{2014} + 1)^4$.

(a) 10
(b) 12
(c) 14
(d) 16
(e) None of these

Problem 8

(LF 2014 9-10)

The digit sum of a number is the sum of its decimal digits. For example, the digit sum of the number 3206 is $3 + 2 + 0 + 6 = 11$. Determine the digit sum of the number $(10^{2014} + 1)^4$.

(a) 10
(b) 12
(c) 14
(d) 16
(e) None of these

Answer: (d)

Final Thoughts

Well, we survived another day. Any thoughts?

Any questions about the contests? About grading? About anything?

Thanks for Participating!