Math Field Day
Prep Session
Grades 9-10

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Alison Garcia

Ashley Lopez

Maria Nogin

California State University, Fresno

March 7, 2019

Formulas you should know

Perimeter, area, surface area, volume:

- triangle
- rectangle
- trapezoid
- parallelogram
- circle
- prism
- pyramid
- cylinder
- cone
- sphere

Similar figures/solids and their perimeters/areas/volumes.

Theorems about angles

Sum of interior angles in any triangle is $180°$, in any n-gon $(n-2) \cdot 180°$.

Relationships between interior/exterior angles in a triangle.

In a regular n-gon, each exterior angle is $\frac{360°}{n}$, each interior angle is $\frac{n-2}{n} \cdot 180°$.

Theorems about triangles

Ratios of lengths of sides of $30° - 60° - 90°$ and $45° - 45° - 90°$ triangles.

The area of an equilateral triangle with side s is $\frac{\sqrt{3}}{4}s^2$.

Pythagorean Theorem.

The three medians in any triangle are concurrent and each median is divided by the intersection point into two parts whole lengths have ratio $1 : 2$.

Example 1

The radius of a sphere is tripled, by what number is its volume multiplied?

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One sheet of metal can be melted down to make a spherical ball with a radius of 2 cm. How many such sheets would need to be melted down to make a spherical ball of radius 6 cm?

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The radius of a sphere is tripled, by what number is its volume multiplied?

One sheet of metal can be melted down to make a spherical ball with a radius of 2 cm. How many such sheets would need to be melted down to make a spherical ball of radius 6 cm?

Solution. Since the volume grows proportionally to the cube of the radius, the volume increases by a factor of $3^3 = 27$ when the radius increases by a factor of 3.

Example 2

What is the measure of each interior angle of a regular decagon?

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What is the measure of each interior angle of a regular decagon?

Solution. Each exterior angle is $\frac{1}{10} \cdot 360^\circ = 36^\circ$, so each interior angle is $180^\circ - 36^\circ = 144^\circ$.

Mad Hatter

- Solo competition, rapid fire computation
- Multiple choice format, problems read aloud
- 2 minutes per problem
- 2 parts with 60 minutes each
- Pencil/paper allowed
- Correct = 1 points, blank = 0 points, incorrect = 0 points
- Highest score WINS

For this mock test, 10 questions will be given. 2 minutes per problem.

Problem 1

If the measure, in degrees, of the three angles of a triangle are x, $x + 10$, and $2x - 6$, the triangle must be

(a) right.

(b) equilateral.

(c) isosceles.

(d) scalene.

Problem 2

The perimeter of a rhombus is 200 feet and one of its diagonals is 80 feet. What is the area of the rhombus?

(a) 1200
(b) 1500
(c) 2000
(d) 2400

Problem 3

An analog clock displays the time 3:40. What is the measure of the smaller angle formed by the minute and hour hands of the clock?

(a) 100°
(b) 110°
(c) 120°
(d) 130°

Problem 4

Three balls are stacked in a cylinder that touches the stack on all sides and on the top and bottom. What is the ratio of the volume of balls to the volume of the cylinder?

(a) $\frac{2}{9}$

(b) $\frac{2}{3}$

(c) $\frac{4}{9}$

(d) $\frac{4}{3}$

Problem 5

The surface area of a large cube is 5400 square inches. This cube is cut into a number of identical smaller cubes, each having a volume of 216 cubic inches. How many smalller cubes are there?

(a) 180
(b) 164
(c) 125
(d) 64

Problem 6

Let BE be a median of triangle ABC, and let D be a point on AB such that $\frac{BD}{DA} = \frac{3}{7}$. What is the ratio of the area of triangle BED to that of triangle ABC?

(a) $\frac{3}{10}$

(b) $\frac{10}{3}$

(c) $\frac{3}{20}$

(d) $\frac{10}{6}$

Problem 7

Three identical coins of radius 1 are placed on a table so that they are mutually tangent. A smaller coin is placed between them tangent to all three. What is the radius of the smaller coin?

(a) $\frac{1}{3}$

(b) $\frac{2}{\sqrt{3}} - 1$

(c) $\sqrt{2} - 1$

(d) $\frac{1}{2\sqrt{3}}$

Problem 8

Two points A and B lie on a sphere of radius 12. The length of the straight line segment joining A and B is $12\sqrt{3}$. What is the length of the shortest path from A to B if every point of the path must lie on the sphere?

(a) 6π

(b) 8π

(c) 9π

(d) 12π

Problem 9

A paper cone has height 12 inches and the diameter of the base has length 10 inches. The cone is cut along one side and unrolled to form a portion of a disk. What angle of the circle does this portion include?

(a) $\frac{5\pi}{13}$

(b) $\frac{5\pi}{12}$

(c) $\frac{10\pi}{13}$

(d) $\frac{5\pi}{6}$

Problem 10

Consider a triangular pyramid $ABCD$ with equilateral base ABC of side length 1 such that $AD = BC = CD$ and $m\angle ADB = m\angle BDC = m\angle ADC = 90°$. Find the volume of $ABCD$.

(a) $\frac{2}{24}$

(b) $\frac{\sqrt{3}}{24}$

(c) $\frac{1}{12}$

(d) $\frac{\sqrt{2}}{24}$

Test Complete!

Take a moment, breathe, relax.

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Answer: (d)

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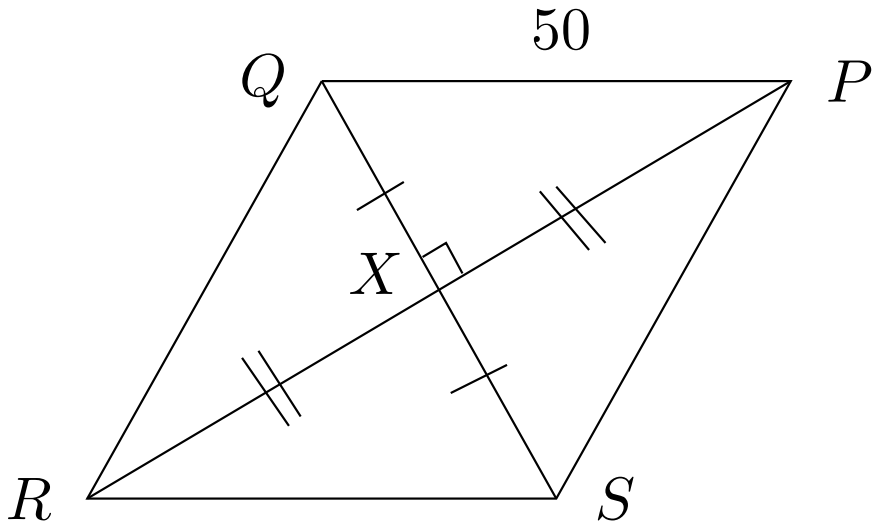

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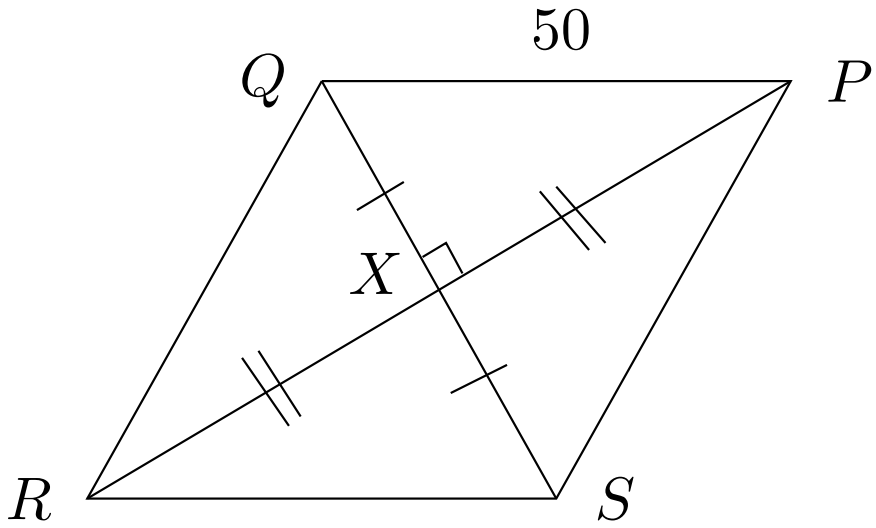

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Answer: (b)

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(a) 180
(b) 164
(c) 125
(d) 64

Answer: (c)

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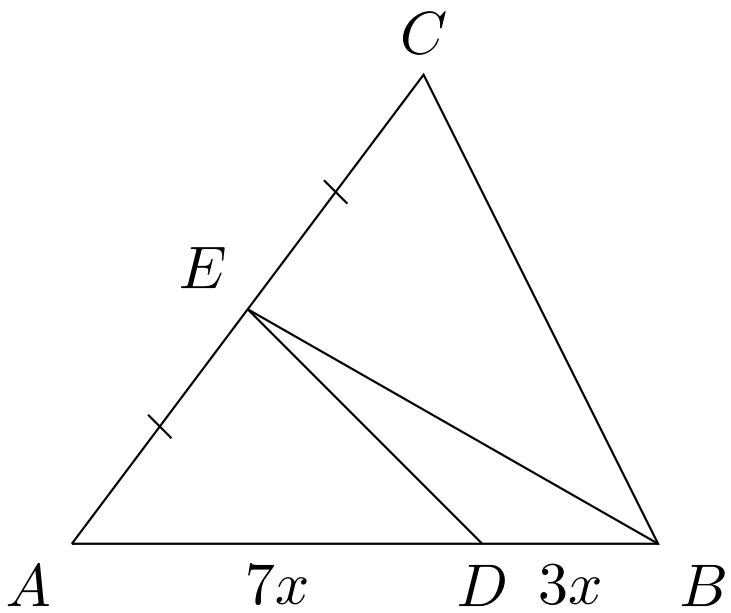

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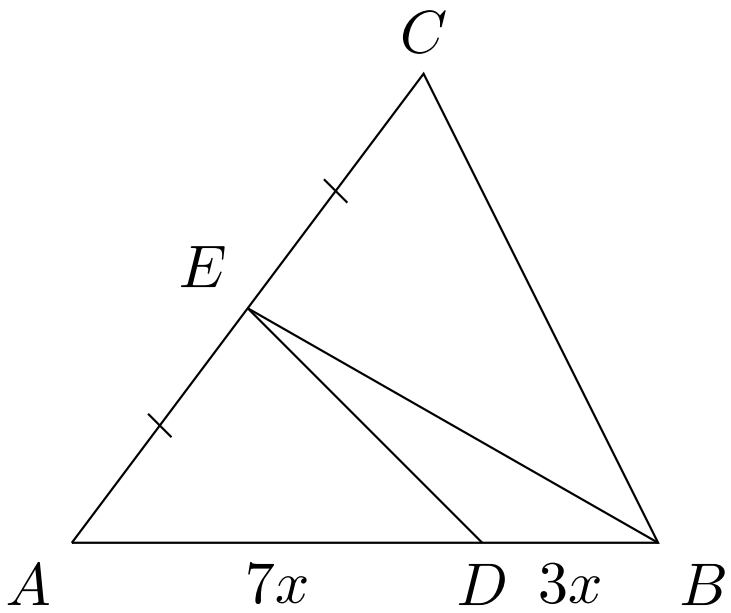

Answer: (c)

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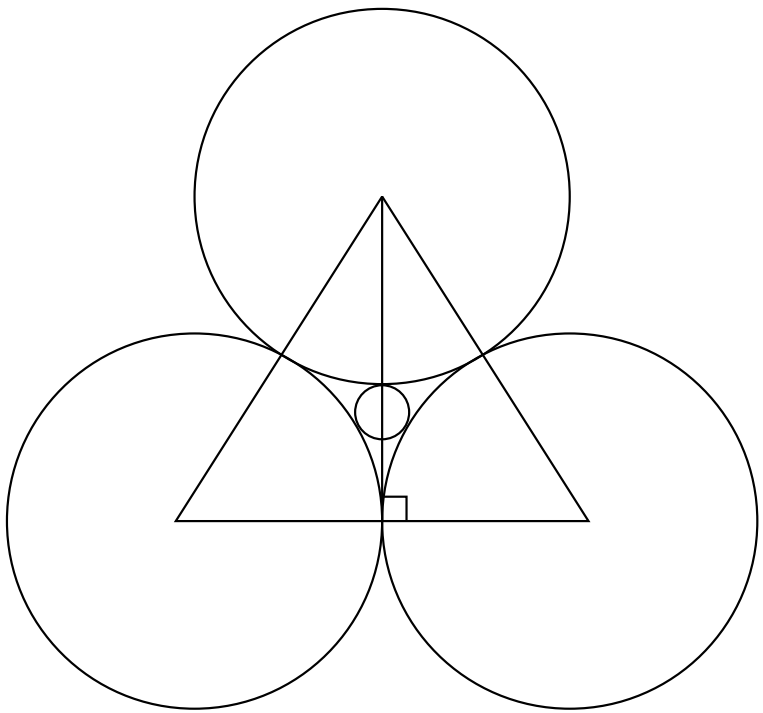

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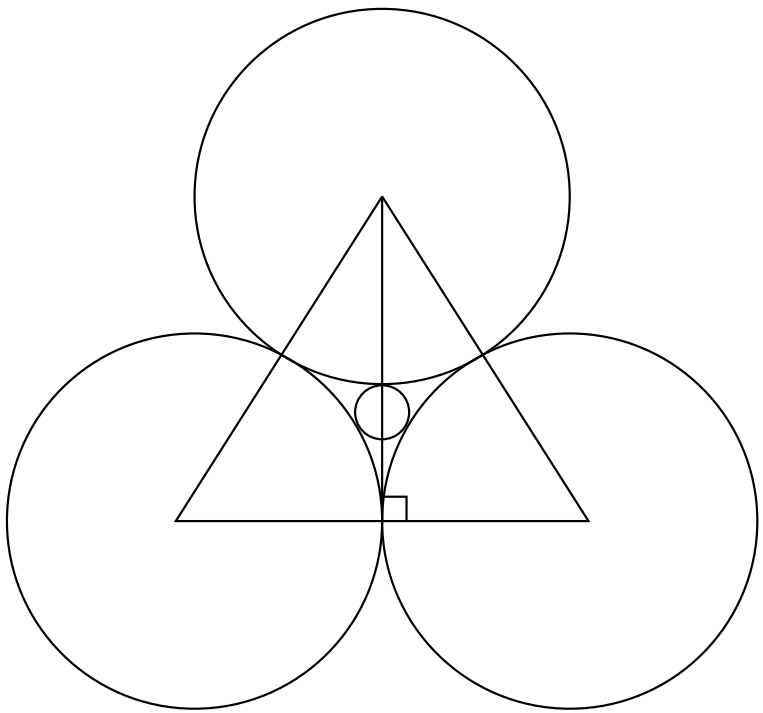
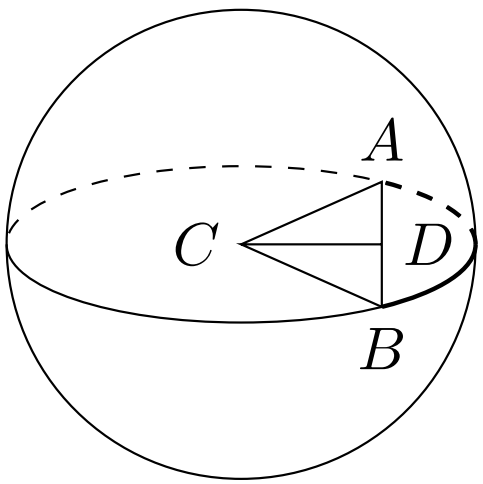
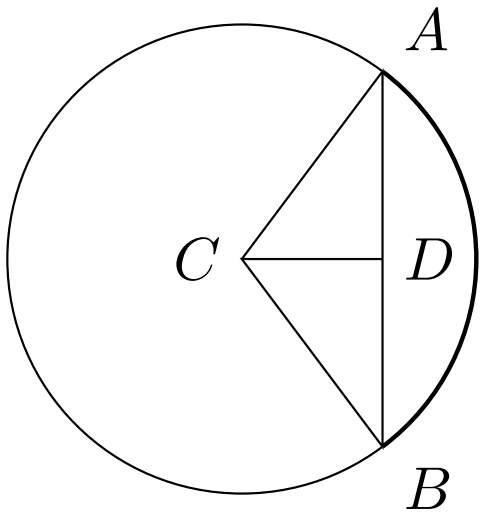


Answer: (b)

Problem 8

(MH 9-10, 2010) Two points A and B lie on a sphere of radius 12. The length of the straight line segment joining A and B is $12\sqrt{3}$. What is the length of the shortest path from A to B if every point of the path must lie on the sphere?





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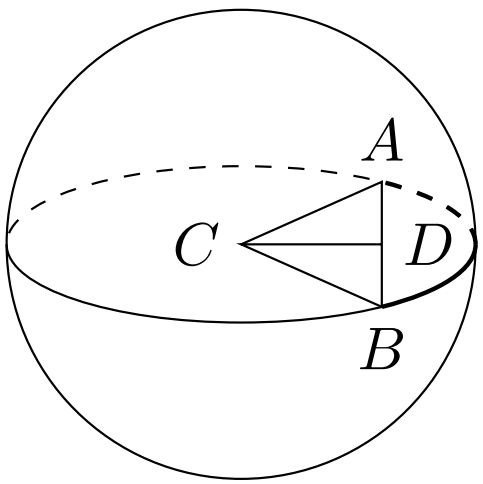

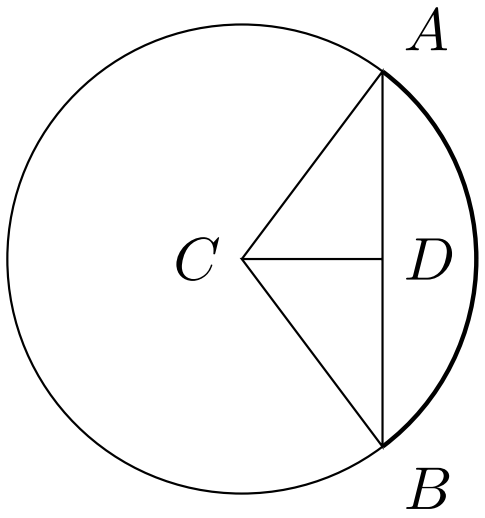

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Answer: (b)

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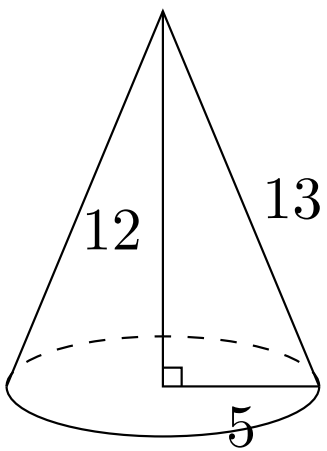

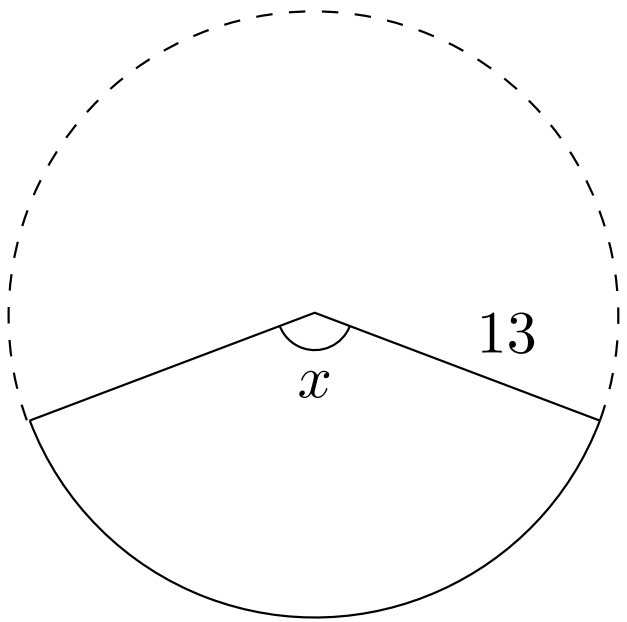

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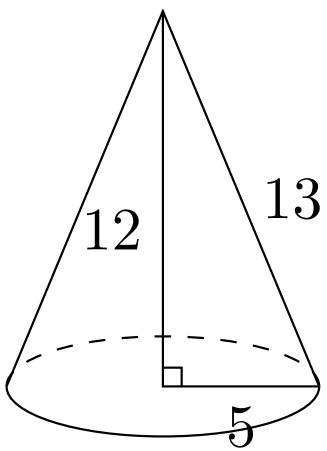

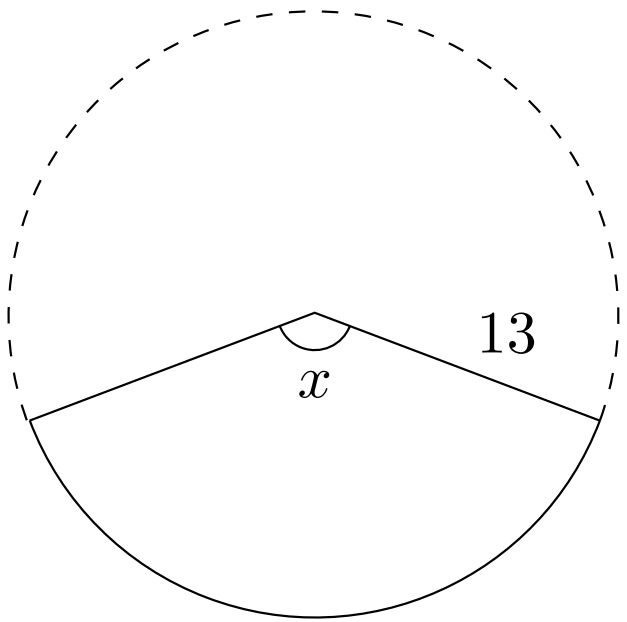

Answer: (c)

Problem 10

(MH 9-10, 2017) Consider a triangular pyramid $ABCD$ with equilateral base ABC of side length 1 such that $AD = BC = CD$ and $m\angle ADB = m\angle BDC = m\angle ADC = 90°$. Find the volume of $ABCD$.

(a) $\frac{2}{24}$

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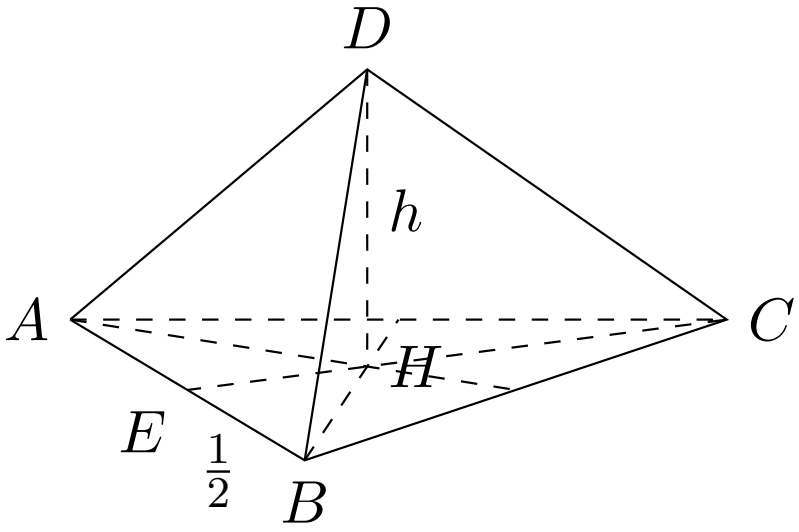

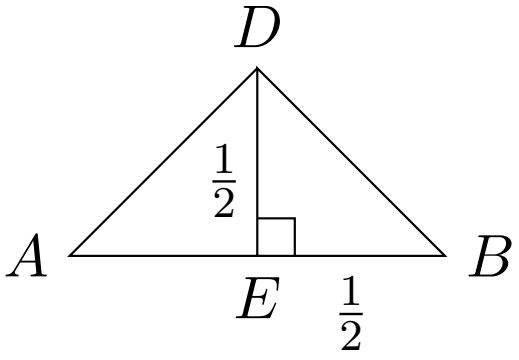

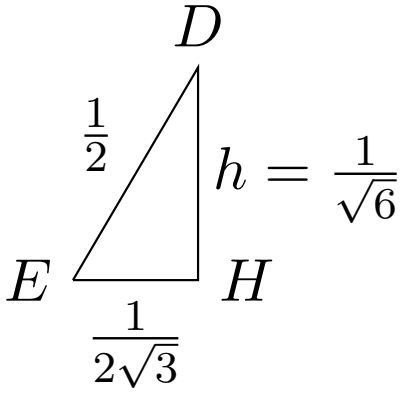

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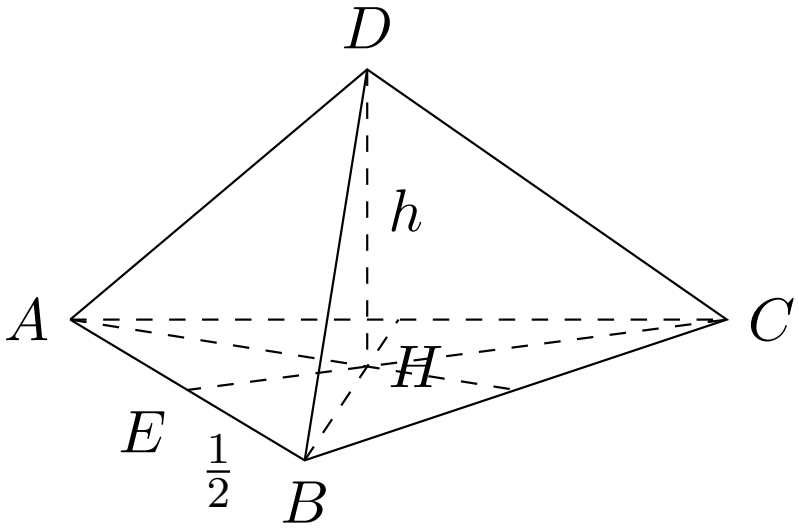
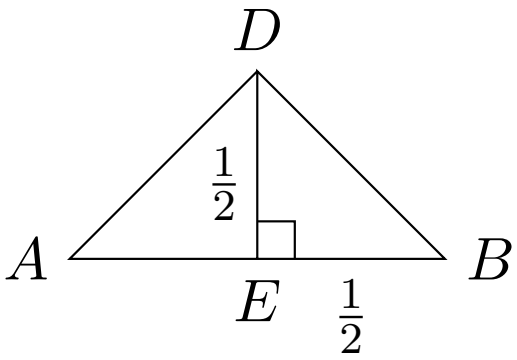
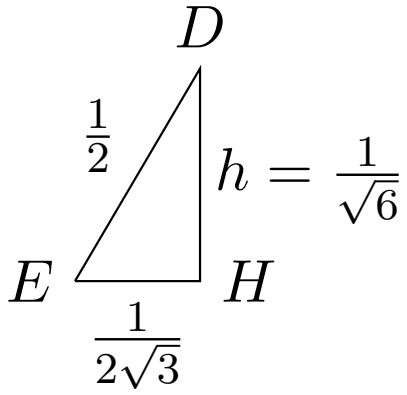






Answer: (d)

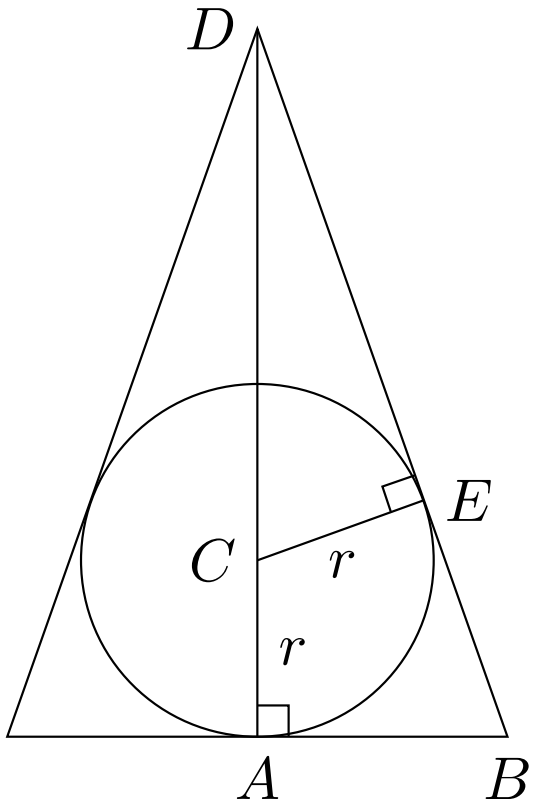
Leap Frog

- Teams of two people from the same school
- Each team member gets 10 problems
- First hour: each team member works on his/her own 10 problems, no communication is allowed
- Second hour: may communicate, exchange papers, etc.
- Turn in a single set of 20 answers from the team
- Correct $= 4$ points, blank $= 0$ points, incorrect $= -1$ points

For this mock test, each team member will get 4 problems.
20 minutes per part.

Problem 1

(LF 9-10, 2017) A circle is inscribed in the isosceles triangle with respective side lengths 6, 6 and 4. Determine the area of the inscribed circle.



(a) $\pi/2$
(b) $3\pi/2$
(c) $5\pi/2$
(d) $7\pi/2$
(e) None of these

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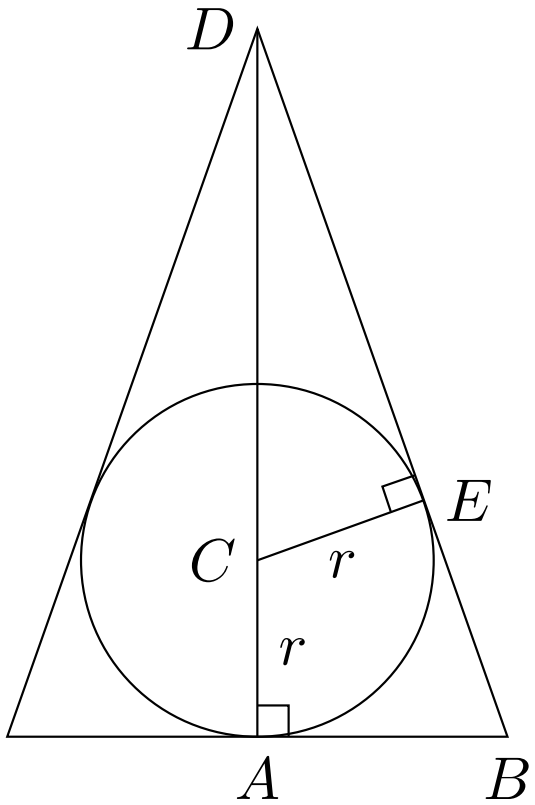
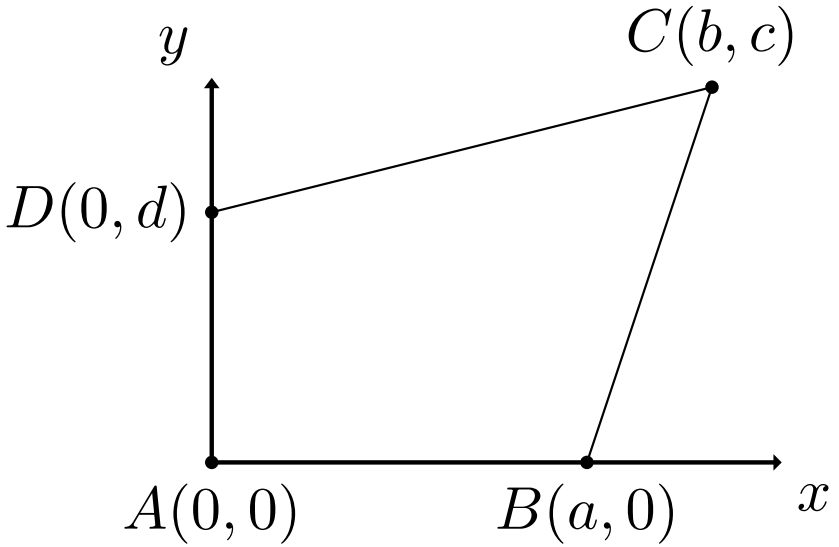


(a) $\pi/2$
(b) $3\pi/2$
(c) $5\pi/2$
(d) $7\pi/2$
(e) None of these

Answer: (e)

Problem 2

(LF 9-10, 2015) Quadrilateral $ABCD$ in the Cartesian plane is pictured below. Determine the area enclosed by $ABCD$. (You may assume $b > a$ and $c > d$ as pictured.)



(a) Area $= \frac{1}{4}(a+b)(d+c)$

(b) Area $= \frac{1}{4}(a+d)(b+c)$

(c) Area $= \frac{1}{2}(ad+bc)$

(d) Area $= \frac{1}{2}(ac+bd)$

(e) None of these

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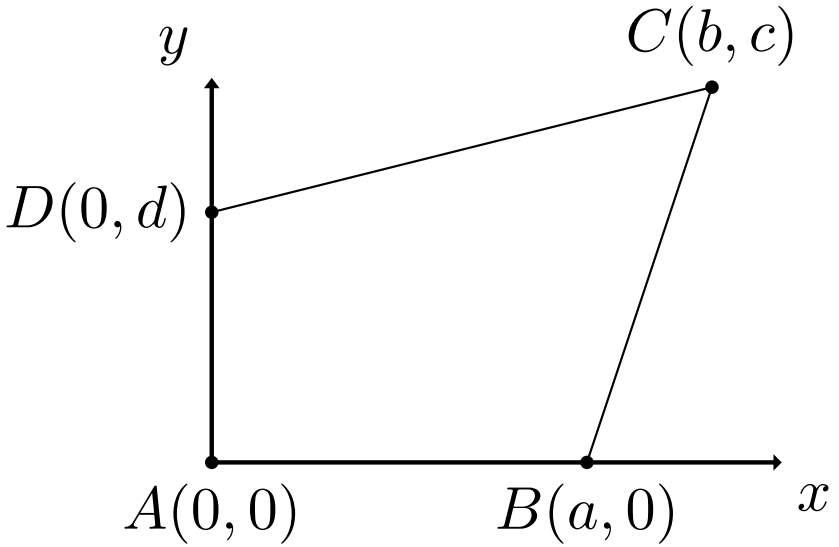

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(c) Area $= \frac{1}{2}(ad+bc)$

(d) Area $= \frac{1}{2}(ac+bd)$

(e) None of these

Answer: (d)

Problem 3

(LF 9-12, 2005) What is the volume of the cube that circumscribes the sphere that circumscribes the cube that circumscribes the sphere of radius 1 inch?

(a) $9\sqrt{3}$ in^3
(b) $16\sqrt{2}$ in^3
(c) $24\sqrt{3}$ in^3
(d) $54\sqrt{2}$ in^3
(e) None of these

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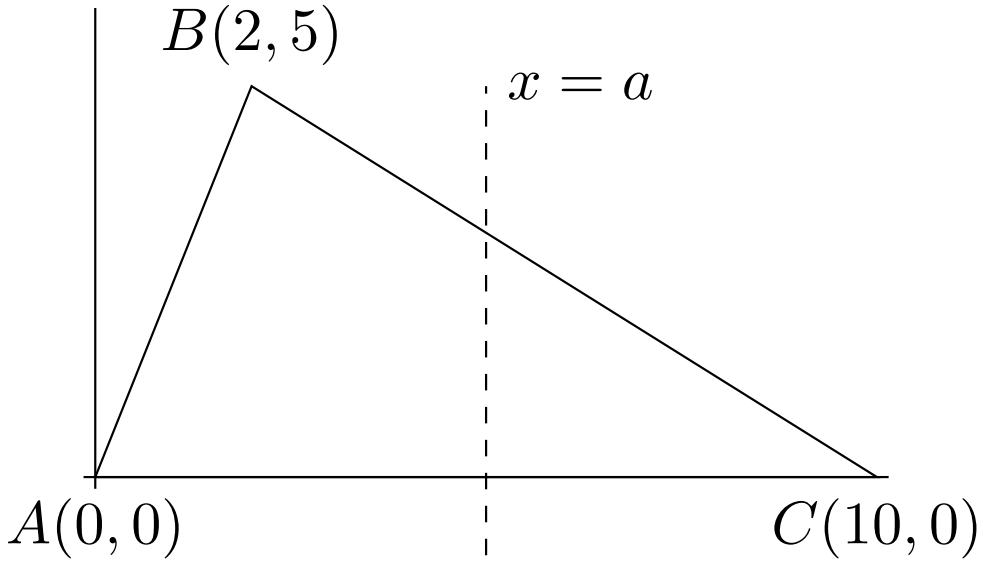
(c) $24\sqrt{3}$ in^3
(d) $54\sqrt{2}$ in^3
(e) None of these

Answer: (c)

Problem 4

(LF 9-10, 2015) What is the value of a so that the vertical line $x = a$ divides the triangle $\triangle ABC$ pictured below into two regions of equal area?



(a) $a = \sqrt{7}$

(b) $a = \dfrac{7}{2}$

(c) $a = 3$

(d) $a = 10 - 2\sqrt{10}$

(e) None of these

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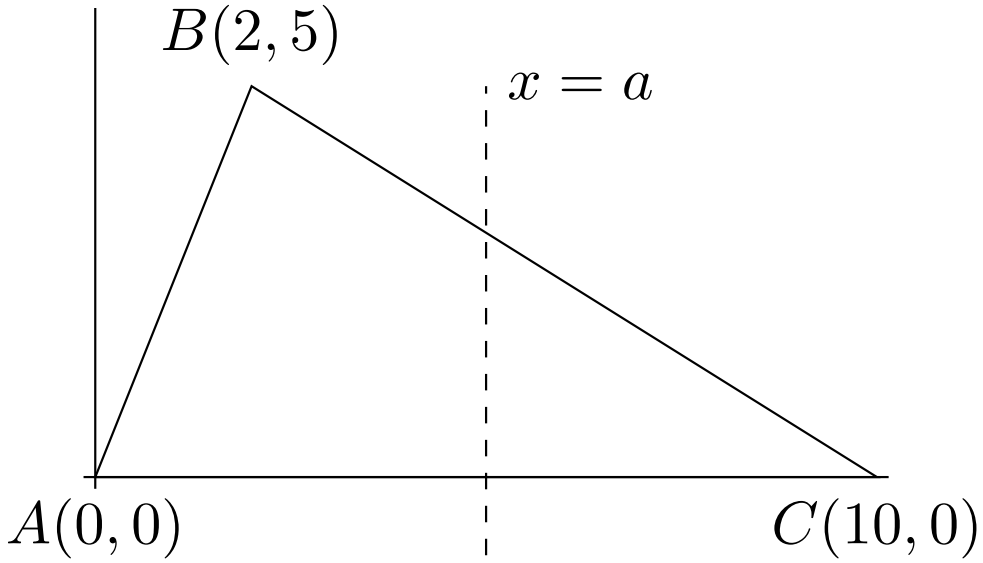

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(d) $a = 10 - 2\sqrt{10}$

(e) None of these

Answer: (d)

Problem 5

(LF 9-10, 2015) In the figure below, the rectangle is a square, whose side lengths are all equal to the value a, and the circle is inscribed as pictured. Determine the radius, r, of the inscribed circle.

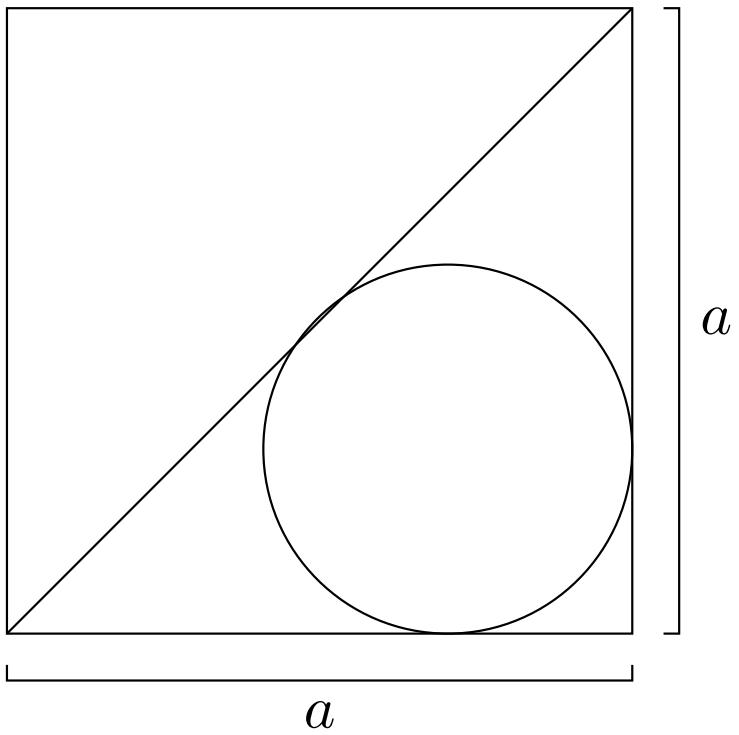
(a) $r = a\left(\dfrac{\sqrt{2}}{2}\right)$

(b) $r = a\left(1 - \dfrac{\sqrt{2}}{2}\right)$

(c) $r = a(\sqrt{2} - 1)$

(d) $r = a(2 - \sqrt{2})$

(e) None of these



Problem 5

(LF 9-10, 2015) In the figure below, the rectangle is a square, whose side lengths are all equal to the value a, and the circle is inscribed as pictured. Determine the radius, r, of the inscribed circle.

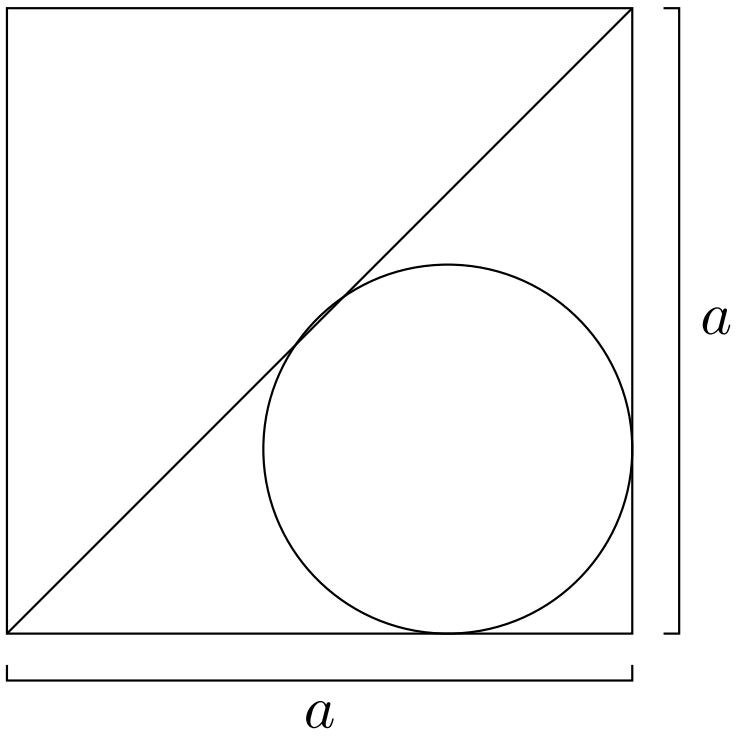
(a) $r = a\left(\frac{\sqrt{2}}{2}\right)$

(b) $r = a\left(1 - \frac{\sqrt{2}}{2}\right)$

(c) $r = a(\sqrt{2} - 1)$

(d) $r = a(2 - \sqrt{2})$

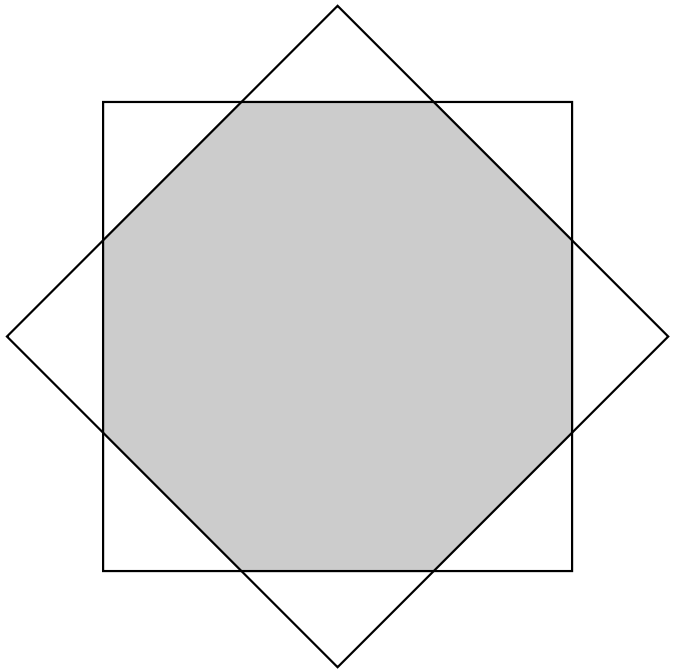
(e) None of these



Answer: (b)

Problem 6

(LF 9-10, 2015) Two $2' \times 2'$ squares share the same center and one square is rotated $45°$ with respect to the other square (see picture below). Determine the shaded area that is enclosed by both squares.



(a) $4\sqrt{2} - 4$ ft^2

(b) $4\sqrt{2} + 4$ ft^2

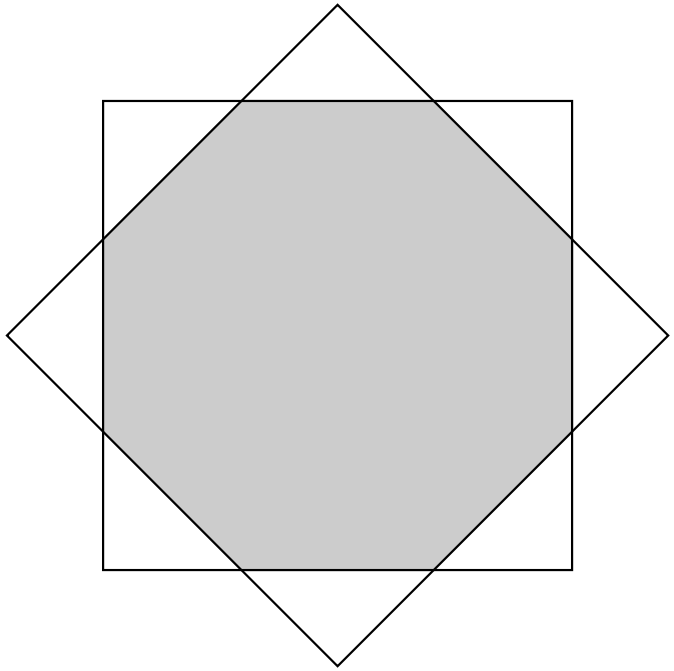
(c) $2\sqrt{2} + 2$ ft^2

(d) $8\sqrt{2} - 8$ ft^2

(e) None of these

Problem 6

(LF 9-10, 2015) Two $2' \times 2'$ squares share the same center and one square is rotated $45°$ with respect to the other square (see picture below). Determine the shaded area that is enclosed by both squares.

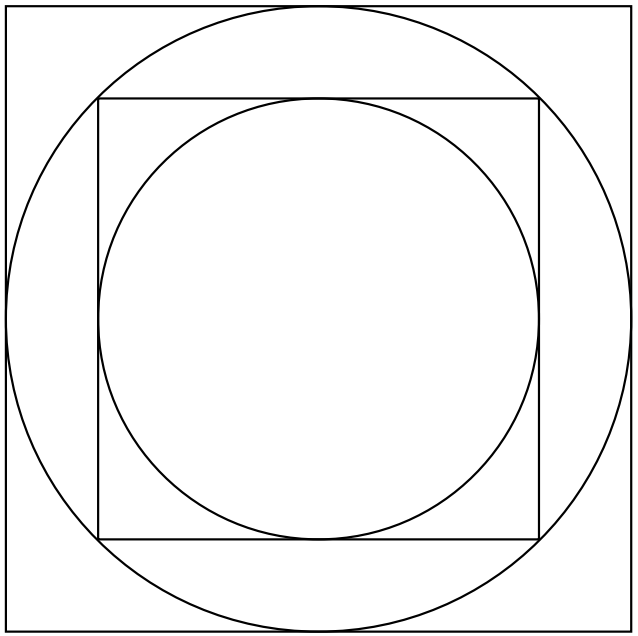


(a) $4\sqrt{2} - 4$ ft^2
(b) $4\sqrt{2} + 4$ ft^2
(c) $2\sqrt{2} + 2$ ft^2
(d) $8\sqrt{2} - 8$ ft^2
(e) None of these

Answer: (d)

Problem 7

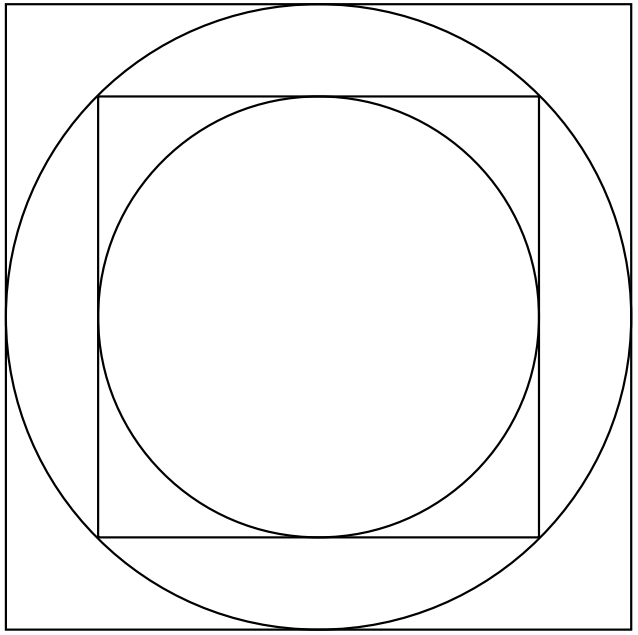
(LF 9-10, 2017) A circle is inscribed in a square. A square is inscribed in that circle. A second circle is inscribed in that square. What is the ratio of the area of the smallest circle to the area of the largest square?



(a) $\pi/2$
(b) $\pi^2/4$
(c) $\pi/8$
(d) $\pi^2/16$
(e) None of these

Problem 7

(LF 9-10, 2017) A circle is inscribed in a square. A square is inscribed in that circle. A second circle is inscribed in that square. What is the ratio of the area of the smallest circle to the area of the largest square?



(a) $\pi/2$
(b) $\pi^2/4$
(c) $\pi/8$
(d) $\pi^2/16$
(e) None of these

Answer: (c)

Problem 8

(MH 9-10, 2010) A cylinder with radius r and height h has volume 1 and total surface area 12. Compute $\frac{1}{r} + \frac{1}{h}$.

(a) $\frac{1}{12}$

(b) $\frac{1}{6}$

(c) 6

(d) 12

(e) None of these

Problem 8

(MH 9-10, 2010) A cylinder with radius r and height h has volume 1 and total surface area 12. Compute $\frac{1}{r} + \frac{1}{h}$.

(a) $\frac{1}{12}$

(b) $\frac{1}{6}$

(c) 6

(d) 12

(e) None of these

Answer: (c)

Final Thoughts

This was our second practice session. Any thoughts?

Any questions about the contests? About grading? About anything?

Thanks for Participating!

Next session: Thursday, April 4, 5:30-8:00, PB 138