Hoover High School Math League

March 25-26, 2009

Coordinate Geometry: Problems

1. Find the distance between $(2,-1)$ and $(7,4)$.
   (a) 6
   (b) $\sqrt{50}$
   (c) $\sqrt{130}$
   (d) 12
   (e) None of the above
2. The equation of the line parallel to the line $4y-x=20$ and containing the point $(2,-3)$ is
   (a) $y=4x-7$
   (b) $y=\frac{1}{4}x-\frac{7}{2}$
   (c) $y=\frac{3}{4}x+\frac{7}{2}$
   (d) $y=-4x+5$
   (e) None of the above
3. Which of the following statements describes the graph of $f(x)=x^2-18x-1$?
   (a) parabola with vertex $(-9,242)$
   (b) parabola with vertex $(9,-82)$
   (c) parabola with vertex $(0,0)$
   (d) not a parabola
   (e) None of the above
4. The graph of an equation $x^2+y^2+4y=14x+11$ is
   (a) a circle
   (b) a point
   (c) an ellipse
   (d) a parabola
   (e) None of the above
5. The equation of a circle is $x^2+y^2+8x-2y+15=0$.
   (a) The center is $(-4,1)$ and the radius is $\sqrt{2}$.
   (b) The center is $(7,-2)$ and the radius is 8.
   (c) The center is $(4,3)$ and the radius is $\sqrt{5}$.
   (d) The center is $(-7,1)$ and the radius is 9.
   (e) None of the above

6. The x-intercept of $3y-3x-8=0$ is
   (a) $\frac{8}{3}$
   (b) $\frac{3}{8}$
   (c) $\frac{-8}{3}$
   (d) $\frac{-3}{8}$
7. The point (a,b) is reflected over the y-axis to the point (c,d) which is reflected over the x-axis to the point (e,f). What is $ab-ef$?
   (a) 2
   (b) $2ab$
   (c) 0
   (d) none of the above
8. Three vertices of parallelogram $ABCD$ are $A(-1,1)$, $B(4,5)$, and $C(3,1)$. Find the coordinates of the fourth vertex D.
   (a) $(-3,-4)$
   (b) $(-2,-3)$
   (c) $(1,1)$
   (d) $(7,0)$
   (e) None of the above
9. A man travels 2 miles north, 2 miles east, one mile south, one mile west, 3 miles north, and 3 miles east. How far is he from the starting point?
   (a) $2\sqrt{4}$ miles
   (b) 6 miles
   (c) $4\sqrt{2}$ miles
   (d) none of the above
10. A line through the points $(m,-9)$ and $(7,m)$ has slope m. What is the value of m?
    (a) 1
    (b) 2
    (c) 3
    (d) 4
    (e) 5
11. Determine the point(s) of intersection (if any) of the line $x+y=2$ with the curve defined by $x^2-y^2=4$.
    (a) The line does not intersect the curve.
    (b) The line intersects the curve at the two points: $(2,0)$ and $(0,-2)$.
    (c) The line intersects the curve at the two points: $(-2,0)$ and $(2,0)$.
    (d) The line intersects the curve at the one point: $(2,0)$.
    (e) None of the above.

12. Given a line segment with endpoints $(-3,4)$ and $(-12,16)$, determine the coordinates of a point on the line whose distance from the right endpoint is one-third the length of the line segment.
    (a) $(2,9)$
    (b) $(-3+\sqrt{41},4+\sqrt{41})$
    (c) $(-6,8)$
    (d) $(5,-20/3)$
    (e) None of the above
13. If the function $f(x) = ax^2 + 3x - 8$ has a minimum value at $x = -2$, then
    (a) $a = 5$
    (b) $a = \frac{7}{9}$
    (c) $a = \frac{3}{4}$
    (d) $a = 12$
    (e) None of the above
14. What is the oblique asymptote of $f(x) = \frac{3x^2-7x+2}{x-2}$?
    (a) $y = 3x+1$
    (b) $y = 2x+1$
    (c) $y = 3x-2$
    (d) $y = 3x-1$
    (e) None of the above
15. Determine the equation of the circle centered at $(-1,1)$ and tangent to the line $y = 5$.
    (a) $(x-1)^2+(y+1)^2 = 16$
    (b) $(x-1)^2+(y+1)^2 = 25$
    (c) $(x-1)^2+(y+1)^2 = 36$
    (d) $(x+1)^2+(y-1)^2 = 16$
    (e) $(x+1)^2+(y-1)^2 = 36$
16. Which of the following is the equation of a parabola with a maximum at $(-1,2)$ and passing through $(2,-1)$?
    (a) $(y-2) = -\frac{1}{3}(x+1)^2$
    (b) $(y+2) = (x-1)^2$
    (c) $(y+2) = -\frac{1}{3}(x+1)^2$
    (d) $(y-2) = -3(x+1)^2$
    (e) $(y-2) = 3(x+1)^2$

17. The graph of $y^2 = 2x^2 + 5x - 3$ is:
    (a) symmetric about the y-axis
    (b) symmetric about the x-axis
    (c) symmetric about the origin
    (d) is not symmetric about any line
    (e) None of the above
18. The slope of the line that goes through the point $(2,0)$ and is tangent to the circle $x^2 + y^2 = 1$ in the first quadrant is
    (a) $\frac{-1}{3}$
    (b) $\frac{-1}{2}$
    (c) $\frac{-1}{\sqrt{3}}$
    (d) $\frac{-1}{\sqrt{2}}$
    (e) None of these
19. The graphs of the lines $y = x - 2$ and $y = mx + 3$ intersect at a point whose x-coordinate and y-coordinate are both positive if and only if
    (a) $m < 1$
    (b) $m = 1$
    (c) $-\frac{3}{2} < m < 0$
    (d) $-\frac{3}{2} < m$
    (e) $-\frac{3}{2} < m < 1$
20. Among all real number pairs (x,y) that satisfy $x^2 + x + y^2 + y = 1$, find the largest possible value of $x + y$.
    (a) $\sqrt{2} - 1$
    (b) 1
    (c) $\sqrt{3} - 1$
    (d) $\sqrt{3}$
    (e) None of these
21. For the ellipse $4x^2 + 9y^2 - 16x + 18y - 11 = 0$
    (a) The center is $(2,-1)$ and the foci are $(2 \pm \sqrt{5}, -1)$
    (b) The center is $(4,1)$ and the foci are $(2 \pm \sqrt{5}, -1)$
    (c) The center is $(2,-1)$ and the foci are $(3 \pm \sqrt{6}, -1)$
    (d) The center is $(4,9)$ and the foci are $(3 \pm \sqrt{6}, -1)$
    (e) None of the above

22. Suppose the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ can be inscribed in the diamond shape whose vertices are $(1,0)$, $(0,1)$, $(-1,0)$, $(0,-1)$. Then $a^2+b^2=$

    (a) 1

    (b) a^2b^2

    (c) $\frac{1}{ab}$

    (d) a^4b^4

    (e) None of the above

23. Determine the equation in rectangular coordinates of $\cos\theta+\sin\theta=1$.

    (a) $x=0$

    (b) $y=0$

    (c) $xy=0$

    (d) The equation cannot be converted to rectangular coordinates

    (e) None of the above

24. Convert the polar equation $r-r\sin\theta=2$ to a rectangular equation.

    (a) $x^2-4y+4=0$

    (b) $x^2+4y+4=0$

    (c) $x^2-2y-2=0$

    (d) $x^2-4y-4=0$

    (e) None of the above

25. Planet M orbits around its sun, S, in an elliptical orbit with the sun at one focus. When M is closest to S, it is 2 million miles away. When M is farthest from S, it is 18 million miles away. Determine the equation of motion of planet M around its sun S, using S as the center of the coordinate plane and assuming the other focus lies on the positive x-axis.

    (a) $\frac{x^2}{100}+\frac{y^2}{36}=1$

    (b) $\frac{x^2}{100}+\frac{y^2}{64}=1$

    (c) $\frac{(x-6)^2}{100}+\frac{y^2}{64}=1$

    (d) $\frac{(x-8)^2}{100}+\frac{y^2}{36}=1$

    (e) $\frac{(x-8)^2}{100}+\frac{(y-6)^2}{36}=1$