Hoover High School Math League

April 15-16, 2009

Counting and Probability — Problems

1. At a sandwich shop there are 2 kinds of bread, 5 kinds of cold cuts, 3 kinds of cheese, and 2 kinds of dressing. How many different sandwiches can be prepared using one kind each of bread, cold cuts, cheese, and dressing?
   (a) 12
   (b) 50
   (c) 42
   (d) 30
   (e) None of the above
2. How many different six-digit numbers can be formed from the set $\{1,2,3,4,5,6,7,8,9\}$ if the digits may be repeated?
   (a) 6×9
   (b) 6^9
   (c) 9^6
   (d) $6+9$
   (e) None of the above
3. An integer between 1 and 1000 (inclusive) is selected at random. Find the probability that the integer is divisible by 5.
   (a) $\frac{1}{10}$
   (b) $\frac{1}{100}$
   (c) $\frac{1}{5}$
   (d) $\frac{1}{1000}$
4. In how many ways can a committee of 4 persons be chosen from a group of 9 persons?
   (a) 9^4
   (b) $\frac{9!}{5!}$
   (c) 126
   (d) 36
   (e) None of the above

5. Consider a group of 20 people. If everyone shakes hands with everyone else, how many handshakes take place?
   (a) 400
   (b) 380
   (c) 200
   (d) 190
   (e) None of the above
6. Three separate awards are to be presented to selected students from a class of 20. How many different outcomes are possible if a student can receive any number of awards?
   (a) 8,000
   (b) 1,140
   (c) 120
   (d) 60
   (e) 27
7. Someone simultaneously flips two coins. What is the probability of seeing exactly two presidents?
   (a) 12%
   (b) 25%
   (c) 30%
   (d) 50%
   (e) None of the above
8. What is the probability of flipping a penny three times and seeing at least one head?
   (a) 25%
   (b) 50%
   (c) 75%
   (d) 87.5%
   (e) None of the above
9. Twelve points are taken on a circle. How many triangles can be made using these points as vertices?
   (a) 110
   (b) 342
   (c) 352
   (d) 345
   (e) None of the above

10. A bag contains 4 red, 2 white, and 3 blue marbles. Two marbles are drawn at random all at once. What is the probability that both are white?
    (a) $\frac{2}{9}$
    (b) $\frac{1}{6}$
    (c) $\frac{1}{36}$
    (d) $\frac{4}{9}$
    (e) None of the above
11. From a group of 5 women and 7 men, how many different committees consisting of 2 women and 3 men can be formed?
    (a) 350
    (b) 792
    (c) 900
    (d) 3,991,680
    (e) None of the above
12. What percentage of the first 1,000 natural numbers have a 2 or a 3 somewhere in them? (Choose the best answer)
    (a) 24%
    (b) 37%
    (c) 49%
    (d) 58%
    (e) None of the above
13. What is the probability of rolling two fair dice and having the sum exceed 5?
    (a) $\approx 20\%$
    (b) $\approx 43\%$
    (c) $\approx 66\%$
    (d) $\approx 83\%$
    (e) None of the above
14. In the game of craps, if a player rolls either a 7 or an 11 the first time s/he rolls the two dice, the player wins. What is the probability of the player winning on the first roll of the dice?
    (a) $\frac{2}{9}$
    (b) $\frac{1}{9}$
    (c) $\frac{1}{3}$
    (d) $\frac{1}{6}$
    (e) $\frac{1}{2}$

15. You roll three fair dice. What is the probability that you will see at least one "tripple" (1-1-1, 2-2-2, 3-3-3, and so on)?
    (a) $\approx 1\%$
    (b) $\approx 3\%$
    (c) $\approx 5\%$
    (d) $\approx 15\%$
    (e) None of the above
16. The first digit of a seven-digit telephone number can never be a 0. How many telephone numbers have increasing digits?
    (a) 28
    (b) 720
    (c) 36
    (d) none of the above
17. From a regular deck of 52 playing cards, you turn over a 7 and then 8. What is the probability that the next card you turn over will be a face card (i.e., J, Q, or K)?
    (a) 12%
    (b) 18%
    (c) 24%
    (d) 30%
    (e) None of the above
18. Suppose you deal two cards from a regular deck of 52 cards. What is the probability that they will all be face cards?
    (a) $\approx 1\%$
    (b) $\approx 3\%$
    (c) $\approx 5\%$
    (d) $\approx 7\%$
    (e) None of the above

19. Two cards are randomly selected from an ordinary 52-card playing deck. What is the probability that they form a blackjack? In other words, what is the probability that one of the cards is an Ace and the other one is either a ten, a jack, a queen, or a king?

    (a) $\frac{\binom{4}{1}\binom{16}{1}}{\binom{52}{2}}$

    (b) $\frac{\binom{4}{1}\binom{4}{1}}{\binom{52}{2}}$

    (c) $\frac{2\binom{4}{1}\binom{16}{1}}{\binom{52}{2}}$

    (d) $\frac{2\binom{4}{1}\binom{4}{1}}{\binom{52}{2}}$

    (e) $\frac{\binom{13}{1}\binom{52}{1}}{\binom{52}{2}}$

20. Someone simultaneously flips three coins. What is the probability of seeing exactly two presidents?

    (a) 25%

    (b) 37.5%

    (c) 50%

    (d) 62.5%

    (e) None of the above

21. Someone simultaneously flips four coins. What is the probability of seeing exactly two presidents?

    (a) 25%

    (b) 37.5%

    (c) 50%

    (d) 62.5%

    (e) None of the above

22. A pressure control apparatus contains three electronic tubes. The apparatus will not work unless all tubes are operative. If the probability of failure of each tube over some time interval is 0.1, what is the probability of failure of the apparatus?

    (a) 99.9%

    (b) 72.9%

    (c) 27.1%

    (d) 0.1%

    (e) None of the above

23. John counted the number of subsets that a set X has, and Mary counted the number of subsets the Y has. If John counted 96 more subsets than Mary, how many elements does X have?

    (a) 5

    (b) 9

    (c) 3

    (d) none of the above

24. In how many distinguishable ways can the letters of the word CINCINNATI be arranged?
    (a) 45,850
    (b) 36,520
    (c) 50,400
    (d) none of the above
25. Find the number of distinguishable permutations of letters in the word TALLAHASSEE.
    (a) 6!
    (b) 2!
    (c) $\frac{11!}{9!}$
    (d) $\frac{11!}{3!2!2!2!}$
    (e) None of the above
26. How many nine-digit social security numbers are there containing three 5's, two 6's, and four 8's?
    (a) $\frac{9!}{5!6!8!}$
    (b) $\frac{9!}{3!2!4!}$
    (c) 9!
    (d) $9 \times 3 \times 2 \times 4$
    (e) None of the above
27. The number $2^7 \times 3^4 \times 5 \times 7^2 \times 11^3$ is divisible by how many perfect squares?
    (a) 36
    (b) 48
    (c) 60
    (d) none of the above