2.7 PROBLEM SOLVING

Polya's Corner

In a certain village $\frac{2}{3}$ of the men are married to $\frac{3}{5}$ of the women. Every one married male is married to one female and no one is married to anyone living outside the village. What portion of the population as a whole is married?

1. Understand the problem: Could a village, such as the one described in the problem, exist? If yes, would the village have more male or more female residents?
2. Devise a plan: Could you use a table or a diagram or a picture to visualize the situation in the village?
3. Carry out the plan: How could you write down the required ratio?
4. Look back: What can you say about the number of people living in the village?

EXERCISES

1. In each of the following parts, write a story problem involving the requested operation.
   (a) fraction addition.
   (b) fraction subtraction.
   (c) fraction multiplication.
   (d) fraction division.
2. Two-thirds of a fish weighs $10\frac{1}{2}$ pounds. How heavy is the whole fish?
3. A suit is on sale for \$180. What was the original price of the suit if the discount was $\frac{1}{4}$ of the original price? Explain how you found your answer and how you can check your answer.
4. James uses $1\frac{1}{2}$ cups of milk and $2\frac{1}{4}$ cups of flour for his favorite cookie recipe. This makes 60 cookies. How much milk and flour would he need to make 40 cookies?
5. Write a story problem to describe each of the following

   (a) $1\frac{2}{5} \times \frac{3}{4}$ **(b)** $\frac{3}{5} \times 1\frac{2}{3}$

   Then choose an appropriate visual model to draw a diagram(s), and explain how to get the answer to your story problem.

6. Show the multiplication of the following numbers using the area model.
   (a) 26×314

   (b) $1\frac{2}{3} \times 2\frac{3}{5}$

   (c) For each computation above write a word problem (two distinct problems) for which you would have to do this calculation to get an answer.

7. What is $1\frac{3}{4} \div \frac{1}{2}$?

   (a) Show your calculation to get the answer.
   (b) Explain (using a picture or diagram or pattern blocks, etc.) to show what the problem and its solution represent.
   (c) Make up a word problem for which you would have to do this calculation to get an answer.

8. Write a story problem using the partitive model to describe

$$1\frac{1}{5} \div \frac{1}{4}$$

9. Write a story problem for division of fractions using the measurement model.

10. Write a story problem for division of fractions using partitive model.

11. Write a story problem for division of fractions using factors and product (area) model.

12. Write a story problem to describe each of the following

    (a) $\frac{1}{3} \div \frac{2}{5}$ **(b)** $\frac{3}{5} \div 1\frac{1}{2}$ **(c)** $\frac{3}{2} \div \frac{3}{4}$

    Then choose an appropriate visual model to draw a diagram(s), and explain how to get the answer to your story problem.

13. Use the area model to represent the multiplication of

$$1\frac{1}{3} \times \frac{3}{4}$$

    and find the answer using the diagram.

14. **Find at least two different solutions for the following problem.**
    The floor of a rectangular room is to be tiled with $\frac{1}{3}$ foot long, square-shaped tiles along a $9\frac{1}{4}$ foot wall. How many tiles will be needed along the wall?

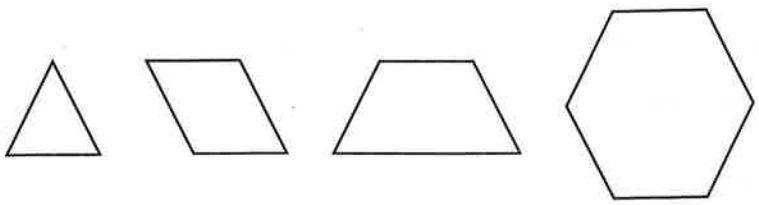
15. **Find at least two different solutions for the following problem.**
    A land developer wants to develop 10 acres of land. Each lot in the development is to be $\frac{2}{9}$ of an acre. How many lots will the land developer have in the 10 acres?

16. Find three fractions such that they are between the following numbers. Then put the five numbers in increasing order. Explain your thinking.

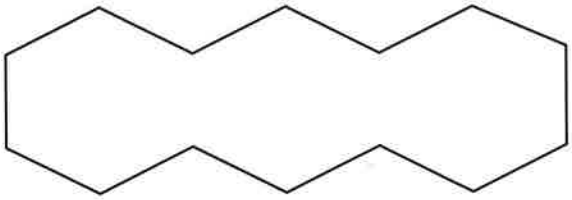
    (a) $\frac{9}{35}$ and $\frac{1}{5}$ **(b)** $\frac{8}{21}$ and $\frac{1}{3}$ **(c)** $\frac{8}{35}$ and $\frac{1}{7}$ **(d)** $\frac{2}{5}$ and $\frac{3}{7}$

17. How many elements are there in the solution sets of the following problems?
    - Problem 1: Find an integer that is more than $\frac{7}{4}$ but less than $\frac{9}{5}$.
    - Problem 2: Find a fraction with denominator 100 that is more than $\frac{7}{4}$ but less than $\frac{9}{5}$.
    - Problem 3: Find a fraction that is more than $\frac{7}{4}$ but less than $\frac{9}{5}$.

18. Consider the pattern blocks and the 'one whole' given below.



Triangle **Parallelogram** **Trapezoid** **Hexagon**



One Whole

(a) What fractions of the whole above are the standard 4 pattern blocks? Write your answers inside the corresponding figures given above.
(b) Represent the following fractions using the whole given above, each in more than one way.

$$\frac{4}{9} =$$

$$\frac{1}{2} =$$

(c) Show how to find the sum $\frac{4}{9} + \frac{1}{2} = \frac{17}{18}$ by manipulating pattern blocks.

19. Consider the pattern blocks and the 'one whole' given in problem 18.
(a) Represent the following fractions using the whole given above, each in more than one way.

$$\frac{5}{6} =$$

$$\frac{1}{2} =$$

(b) Show how to find the sum $\frac{5}{6} - \frac{1}{2} = \frac{2}{6}$ by manipulating pattern blocks.

20. Consider the pattern blocks and the 'one whole' given in problem 18.
(a) Represent the following two fractions using the whole given above, each in more than one way.

$$\frac{4}{9} =$$

$$\frac{1}{3} =$$

(b) Show how to find the sum $\frac{4}{9} + \frac{1}{3} = \frac{7}{9}$ by manipulating pattern blocks.

21. Consider the pattern blocks and the 'one whole' given in problem 18.
(a) Represent the following fractions using the whole given above, each in more than one way.

$$\frac{5}{9} =$$

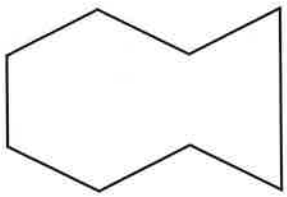
$$\frac{3}{2} =$$

(b) Show how to find the difference $\frac{5}{9} + \frac{3}{2} = 2\frac{1}{18}$ by manipulating pattern blocks.

22. Consider the pattern blocks and the 'one whole' given in problem 18. Use them to compute and explain the operations.

(a) $\frac{1}{6} + 1\frac{3}{4}$ **(b)** $1\frac{5}{6} - \frac{1}{2}$

23. Try to re-do the previous 5 problems but now use the 'one whole' in the following picture. Discuss difficulties and benefits of using this new 'one whole'.



One Whole

24. Using two hexagons as one whole find the following multiplications in two different ways. Repeat this problem using four hexagons as one whole.

(a) $\frac{1}{4} \times \frac{1}{3}$ and $\frac{1}{3} \times \frac{1}{4}$ **(b)** $\frac{3}{4} \times \frac{1}{3}$ and $\frac{1}{3} \times \frac{3}{4}$ **(c)** $\frac{3}{4} \times \frac{2}{3}$ and $\frac{2}{3} \times \frac{3}{4}$

25. Use the number line diagram and fraction rulers to decide which of the following two quantities is larger: three-fourths of 9 or two-thirds of 11.

26. We bought a pizza to share among friends. A third of it is just cheese, a half is pepperoni and the rest is half supreme and half veggie. If the whole pizza was \$24, how much should the vegetarian pay for his slice?

27. Homer has 3 dozen donuts that he wants to share equally among 5 people. How much of a dozen can he give to each person?

What is the 'whole' you used to solve this problem? Can you think of a different 'whole' that solves this problem?

28. President Obama has recently attended a ribbon cutting ceremony and as a souvenir he took $3\frac{3}{4}$ yards of ribbon. He wants to cut it into 3 equal pieces, one for his wife and one for each of his daughters. How long will each piece be?

29. **Find at least two different solutions for the following problem.**
If 0.3 oz of mustard is used on each of 8 thousand hot dogs, how many 12-oz jars of mustard are needed?

30. Bob and Carla bought a pizza. Bob ate a third of the pizza and Carla ate a fourth of the pizza.
    a. How much pizza was left?
    b. If the weight of the remaining pizza is $\frac{5}{9}$ *lb*, what was the weight of the whole pizza?

CHAPTER 2 REFLECTIONS

1. Explain the reason why you need to find a common denominator when you add or subtract fractions.

2. Discuss the use of shortcuts in teaching mathematics (for example, using shortcuts for multi-digit number multiplication). Discuss what the implications in student learning are and what you can do as a future teacher.

3. Explain why any multiplication can be represented as a rectangle. Conjecture where you would find the two numbers you multiply and the product (answer) in that rectangle.

4. Explain why do you multiply by the reciprocal when you have to divide a fraction by another fraction.

5. Explain the reasoning of converting a mixed fraction into an improper fraction.

6. Write about your learning of fractions. It should include the following:
    - Summarize what you have learned about fractions in this section.
    - What new insights do you have about fractions?
    - What is still unclear about fractions?