Measurement and Geometry

1.0 **Students understand the concept of time and units to measure it; they understand that objects have properties, such as length, weight, and capacity, and that comparisons may be made by referring to those properties:**

1.1 Compare the length, weight, and capacity of objects by making direct comparisons with reference objects (e.g., note which object is shorter, longer, taller, lighter, heavier, or holds more).

Who is the tallest girl in the class? The tallest boy?
Which container holds more?

1.2 Demonstrate an understanding of concepts of time (e.g., morning, afternoon, evening, today, yesterday, tomorrow, week, year) and tools that measure time (e.g., clock, calendar).

I left home at 9 o'clock in the morning and came back 4 hours later. Did I come home in the morning or in the afternoon?

1.3 Name the days of the week.

1.4 Identify the time (to the nearest hour) of everyday events (e.g., lunch time is 12 o'clock; bedtime is 8 o'clock at night).

2.0 **Students identify common objects in their environment and describe the geometric features:**

2.1 Identify and describe common geometric objects (e.g., circle, triangle, square, rectangle, cube, sphere, cone).

Which of these is a square?
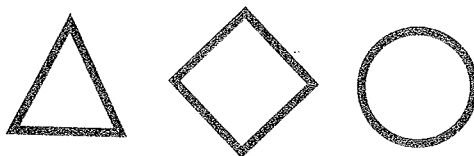
Given 5 squares of the same size, can you make use of some or all of them to form a bigger square?

2.2 Compare familiar plane and solid objects by common attributes (e.g., position, shape, size, roundness, number of corners).

Algebra and Functions

1.0   Students use number sentences with operational symbols and expressions to solve problems:

  1.1   Write and solve number sentences from problem situations that express relationships involving addition and subtraction.

  Do the following problems in succession:

  1.   Marie had some pencils in her desk. She put 5 more in her desk. Then she had 14. How many pencils did she have in her desk to start with?

  2.   Eddie had 14 helium balloons. A number of them floated away. He had 5 left. How many did he lose?

  3.   Nina had 14 seashells. That was 5 more than Pedro had. How many seashells did Pedro have?

  4.   5 + ( ) = 6?   ( ) + 12 = 14?

  1.2   Understand the meaning of the symbols +, −, =.

  1.3   Create problem situations that might lead to given number sentences involving addition and subtraction.

Measurement and Geometry

1.0   Students use direct comparison and nonstandard units to describe the measurements of objects:

  1.1   Compare the length, weight, and volume of two or more objects by using direct comparison or a nonstandard unit.

  Measure your desk by using the length of a ballpoint pen. How many ballpoint pens would be roughly equal to the length of your desk? The width of your desk? Which is longer?

  1.2   Tell time to the nearest half hour and relate time to events (e.g., before/after, shorter/longer).

2.0   Students identify common geometric figures, classify them by common attributes, and describe their relative position or their location in space:

  2.1   Identify, describe, and compare triangles, rectangles, squares, and circles, including the faces of three-dimensional objects.

  Describe the shape of a page in your textbook and compare it to the face of the clock on the wall.

Measurement and Geometry (Continued)

2.2   Classify familiar plane and solid objects by common attributes, such as color, position, shape, size, roundness, or number of corners, and explain which attributes are being used for classification.

2.3   Give and follow directions about location.

Here are pictures on a table of a ball, a girl, a horse, and a cat. Arrange them according to these directions:

1. Put the picture of the ball above the picture of the horse.
2. Put the picture of the girl on top of the picture of the horse.
3. Put the picture of the cat under the picture of the horse.

2.4   Arrange and describe objects in space by proximity, position, and direction (e.g., near, far, below, above, up, down, behind, in front of, next to, left or right of).

Statistics, Data Analysis, and Probability

1.0   Students organize, represent, and compare data by category on simple graphs and charts:

1.1   Sort objects and data by common attributes and describe the categories.

1.2   Represent and compare data (e.g., largest, smallest, most often, least often) by using pictures, bar graphs, tally charts, and picture graphs.

2.0   Students sort objects and create and describe patterns by numbers, shapes, sizes, rhythms, or colors:

2.1   Describe, extend, and explain ways to get to a next element in simple repeating patterns (e.g., rhythmic, numeric, color, and shape).

Mathematical Reasoning

1.0   Students make decisions about how to set up a problem:

1.1   Determine the approach, materials, and strategies to be used.

1.2   Use tools, such as manipulatives or sketches, to model problems.

2.0   Students solve problems and justify their reasoning:

2.1   Explain the reasoning used and justify the procedures selected.

2.2   Make precise calculations and check the validity of the results from the context of the problem.

3.0   Students note connections between one problem and another.

Measurement and Geometry

1.0 **Students understand that measurement is accomplished by identifying a unit of measure, iterating (repeating) that unit, and comparing it to the item to be measured:**

1.1 Measure the length of objects by iterating (repeating) a nonstandard or standard unit.

1.2 Use different units to measure the same object and predict whether the measure will be greater or smaller when a different unit is used.

Four children measured the width of a room by counting how many paces it took them to cross it. It took Ana 9 paces, Erlane 8, Stephen 10, and Carlos 7. Who had the longest pace? (Adapted from TIMSS, gr. 4, L-8; gr. 8, L-12)

Measure the length of your desk with a new crayon and with a new pencil. Which is greater, the number of crayon units or the number of pencil units?

1.3 Measure the length of an object to the nearest inch and/or centimeter.

1.4 Tell time to the nearest quarter hour and know relationships of time (e.g., minutes in an hour, days in a month, weeks in a year).

A bus left the station for the bus barn at 9:45 in the morning. It takes 45 minutes to drive from the station to the bus barn. What time will it be when the bus gets to the bus barn?

Which is a longer period: 3 weeks or 19 days? 27 days or 4 weeks?

1.5 Determine the duration of intervals of time in hours (e.g., 11:00 a.m. to 4:00 p.m.).

2.0 **Students identify and describe the attributes of common figures in the plane and of common objects in space:**

2.1 Describe and classify plane and solid geometric shapes (e.g., circle, triangle, square, rectangle, sphere, pyramid, cube, rectangular prism) according to the number and shape of faces, edges, and vertices.

2.2 Put shapes together and take them apart to form other shapes (e.g., two congruent right triangles can be arranged to form a rectangle).

Algebra and Functions (Continued)

2.2   Extend and recognize a linear pattern by its rules (e.g., the number of legs on a given number of horses may be calculated by counting by 4s or by multiplying the number of horses by 4).

Here is the beginning of a pattern of tiles. Assuming that the pattern continues linearly, how many tiles will be in the sixth figure?
(Adapted from TIMSS gr. 4, K–6)
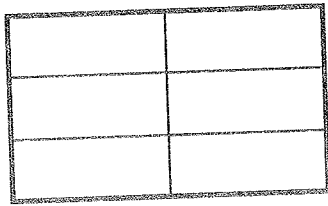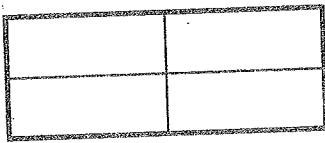
# Measurement and Geometry

1.0   **Students choose and use appropriate units and measurement tools to quantify the properties of objects:**

     1.1   Choose the appropriate tools and units (metric and U.S.) and estimate and measure the length, liquid volume, and weight/mass of given objects.

     1.2   Estimate or determine the area and volume of solid figures by covering them with squares or by counting the number of cubes that would fill them.

     1.3   Find the perimeter of a polygon with integer sides.

     1.4   Carry out simple unit conversions within a system of measurement (e.g., centimeters and meters, hours and minutes).

2.0   **Students describe and compare the attributes of plane and solid geometric figures and use their understanding to show relationships and solve problems:**

     2.1   Identify, describe, and classify polygons (including pentagons, hexagons, and octagons).

Measurement and Geometry (Continued)

2.2 Identify attributes of triangles (e.g., two equal sides for the isosceles triangle, three equal sides for the equilateral triangle, right angle for the right triangle).

2.3 Identify attributes of quadrilaterals (e.g., parallel sides for the parallelogram, right angles for the rectangle, equal sides and right angles for the square).

2.4 Identify right angles in geometric figures or in appropriate objects and determine whether other angles are greater or less than a right angle.

Which of the following triangles include an angle that is greater than a right angle?
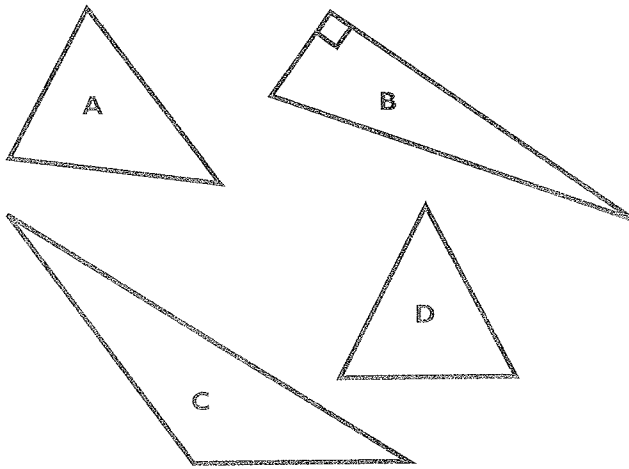
2.5 Identify, describe, and classify common three-dimensional geometric objects (e.g., cube, rectangular solid, sphere, prism, pyramid, cone, cylinder).

2.6 Identify common solid objects that are the components needed to make a more complex solid object.

Measurement and Geometry

1.0   Students understand perimeter and area:

1.1   Measure the area of rectangular shapes by using appropriate units, such as square centimeter (cm^2), square meter (m^2), square kilometer (km^2), square inch (in^2), square yard (yd^2), or square mile (mi^2).

1.2   Recognize that rectangles that have the same area can have different perimeters.

> Draw a rectangle whose area is 120 and whose perimeter exceeds 50. Draw another rectangle with the same area whose perimeter exceeds 240.

1.3   Understand that rectangles that have the same perimeter can have different areas.

> Is the area of a 45×55 rectangle (in cm^2) smaller or bigger than that of a square with the same perimeter?
>
> Draw a rectangle whose perimeter is 40 and whose area is less than 20.

1.4   Understand and use formulas to solve problems involving perimeters and areas of rectangles and squares. Use those formulas to find the areas of more complex figures by dividing the figures into basic shapes.

> The length of a rectangle is 6 cm, and its perimeter is 16 cm. What is the area of the rectangle in square centimeters? (Adapted from TIMSS gr. 8, K–5)

2.0   Students use two-dimensional coordinate grids to represent points and graph lines and simple figures:

2.1   Draw the points corresponding to linear relationships on graph paper (e.g., draw 10 points on the graph of the equation $y = 3x$ and connect them by using a straight line).

2.2   Understand that the length of a horizontal line segment equals the difference of the x-coordinates.

> What is the length of the line segment joining the points $(6, -4)$ and $(21, -4)$?

2.3   Understand that the length of a vertical line segment equals the difference of the y-coordinates.

> What is the length of the line segment joining the points $(121, 3)$ to $(121, 17)$?

Measurement and Geometry (Continued)

3.0 Students demonstrate an understanding of plane and solid geometric objects and use this knowledge to show relationships and solve problems:

3.1 Identify lines that are parallel and perpendicular.

3.2 Identify the radius and diameter of a circle.

3.3 Identify congruent figures.

3.4 Identify figures that have bilateral and rotational symmetry.

Craig folded a piece of paper in half and cut out a shape along the folded edge. Draw a picture to show what the cutout shape will look like when it is opened up and flattened out (Adapted from TIMSS gr. 4, T-5).

Let *AB*, *CD* be perpendicular diameters of a circle, as shown. If we reflect across the line segment *CD*, what happens to *A* and what happens to *B* under this reflection?
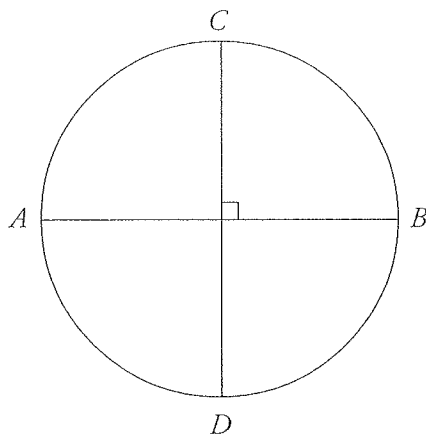
3.5 Know the definitions of a right angle, an acute angle, and an obtuse angle. Understand that 90°, 180°, 270°, and 360° are associated, respectively, with ¼, ½, ¾, and full turns.

3.6 Visualize, describe, and make models of geometric solids (e.g., prisms, pyramids) in terms of the number and shape of faces, edges, and vertices; interpret two-dimensional representations of three-dimensional objects; and draw patterns (of faces) for a solid that, when cut and folded, will make a model of the solid.

3.7 Know the definitions of different triangles (e.g., equilateral, isosceles, scalene) and identify their attributes.

Measurement and Geometry (Continued)

3.8   Know the definition of different quadrilaterals (e.g., rhombus, square, rectangle, parallelogram, trapezoid).

Explain which of the following statements are true and why.

1. All squares are rectangles.

2. All rectangles are squares.

3. All parallelograms are rectangles.

4. All rhombi are parallelograms.

5. Some parallelograms are squares.

Statistics, Data Analysis, and Probability

1.0   Students organize, represent, and interpret numerical and categorical data and clearly communicate their findings:

The following table shows the ages of the girls and boys in a club. Use the information in the table to complete the graph for ages 9 and 10. (Adapted from TIMSS gr. 4, S-1)

Ages	Number of Girls	Number of Boys
8	4	6
9	8	4
10	6	10

Ages of students
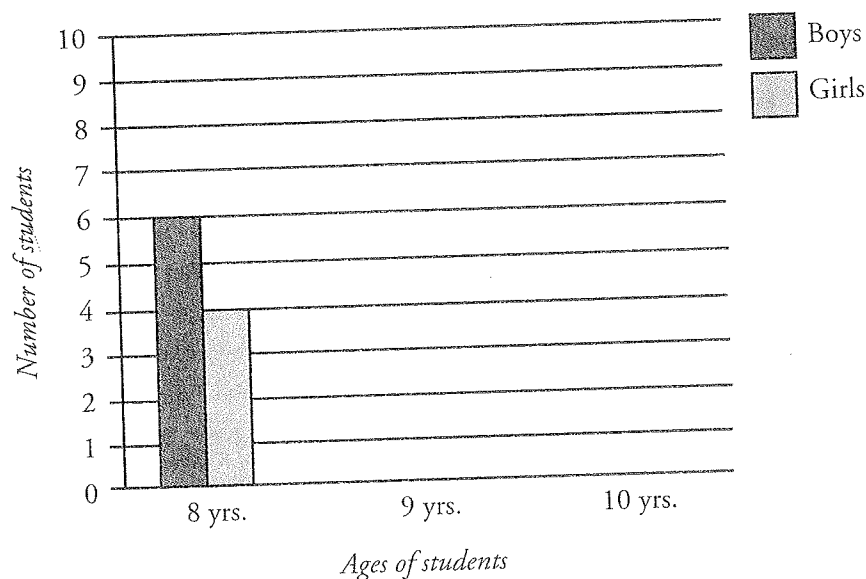
Algebra and Functions

1.0 Students use variables in simple expressions, compute the value of the expression for specific values of the variable, and plot and interpret the results:

1.1 Use information taken from a graph or equation to answer questions about a problem situation.

1.2 Use a letter to represent an unknown number; write and evaluate simple algebraic expressions in one variable by substitution.

$3x + 2 = 14$. What is x?

1.3 Know and use the distributive property in equations and expressions with variables.

1.4 Identify and graph ordered pairs in the four quadrants of the coordinate plane.

Plot the points $(1, 2)$, $(-4, -3)$, $(12, -1)$, $(0, 4)$, $(-4, 0)$.

1.5 Solve problems involving linear functions with integer values; write the equation; and graph the resulting ordered pairs of integers on a grid.

Measurement and Geometry

1.0 Students understand and compute the volumes and areas of simple objects:

1.1 Derive and use the formula for the area of a triangle and of a parallelogram by comparing it with the formula for the area of a rectangle (i.e., two of the same triangles make a parallelogram with twice the area; a parallelogram is compared with a rectangle of the same area by pasting and cutting a right triangle on the parallelogram).

Find the area and perimeter.
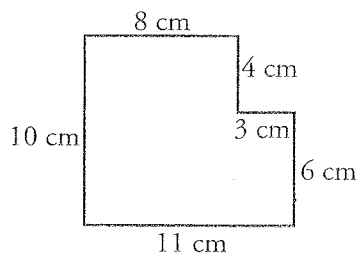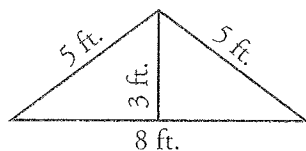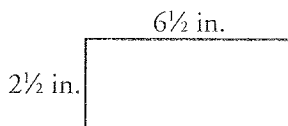
Measurement and Geometry (Continued)

1.2 Construct a cube and rectangular box from two-dimensional patterns and use these patterns to compute the surface area for these objects.

1.3 Understand the concept of volume and use the appropriate units in common measuring systems (i.e., cubic centimeter [cm^3], cubic meter [m^3], cubic inch [$in.^3$], cubic yard [$yd.^3$]) to compute the volume of rectangular solids.

1.4 Differentiate between, and use appropriate units of measures for, two- and three-dimensional objects (i.e., find the perimeter, area, volume).

2.0 Students identify, describe, and classify the properties of, and the relationships between, plane and solid geometric figures:

2.1 Measure, identify, and draw angles, perpendicular and parallel lines, rectangles, and triangles by using appropriate tools (e.g., straightedge, ruler, compass, protractor, drawing software).

2.2 Know that the sum of the angles of any triangle is 180° and the sum of the angles of any quadrilateral is 360° and use this information to solve problems.

2.3 Visualize and draw two-dimensional views of three-dimensional objects made from rectangular solids.

Statistics, Data Analysis, and Probability

1.0 Students display, analyze, compare, and interpret different data sets, including data sets of different sizes:

1.1 Know the concepts of mean, median, and mode; compute and compare simple examples to show that they may differ.

1.2 Organize and display single-variable data in appropriate graphs and representations (e.g., histogram, circle graphs) and explain which types of graphs are appropriate for various data sets.

1.3 Use fractions and percentages to compare data sets of different sizes.

1.4 Identify ordered pairs of data from a graph and interpret the meaning of the data in terms of the situation depicted by the graph.

1.5 Know how to write ordered pairs correctly; for example, (x, y).

Algebra and Functions (Continued)

3.0   Students investigate geometric patterns and describe them algebraically:

3.1   Use variables in expressions describing geometric quantities (e.g., $P = 2w + 2l$, $A = \frac{1}{2} bh$, $C = \pi d$—the formulas for the perimeter of a rectangle, the area of a triangle, and the circumference of a circle, respectively).

A rectangle has width w. Its length is one more than 3 times its width. Find the perimeter of the rectangle. (Your answer will be expressed in terms of w.)

3.2   Express in symbolic form simple relationships arising from geometry.

Measurement and Geometry

1.0   Students deepen their understanding of the measurement of plane and solid shapes and use this understanding to solve problems:

🔵   Understand the concept of a constant such as π; know the formulas for the circumference and area of a circle.

1.2   Know common estimates of π (3.14; $^{22}/_{7}$) and use these values to estimate and calculate the circumference and the area of circles; compare with actual measurements.

1.3   Know and use the formulas for the volume of triangular prisms and cylinders (area of base × height); compare these formulas and explain the similarity between them and the formula for the volume of a rectangular solid.

Find the volumes (dimensions are in cm).
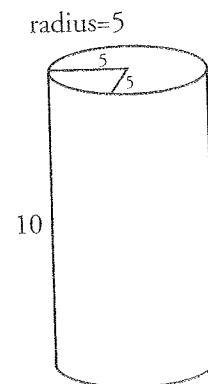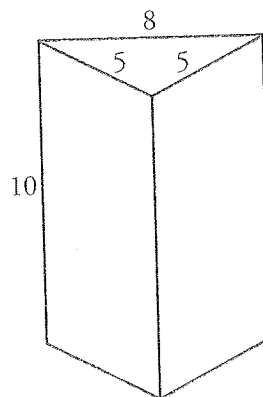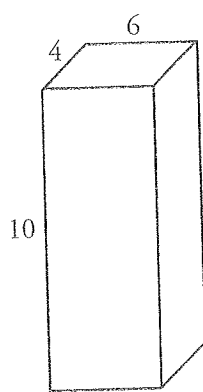
Measurement and Geometry (Continued)

2.0   Students identify and describe the properties of two-dimensional figures:

2.1   Identify angles as vertical, adjacent, complementary, or supplementary and provide descriptions of these terms.

2.2   Use the properties of complementary and supplementary angles and the sum of the angles of a triangle to solve problems involving an unknown angle.

Find the missing angles.
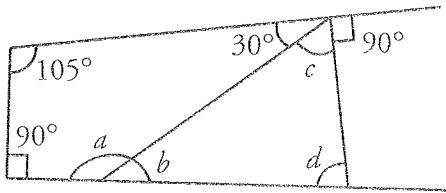
2.3   Draw quadrilaterals and triangles from given information about them (e.g., a quadrilateral having equal sides but no right angles, a right isosceles triangle).

Statistics, Data Analysis, and Probability

1.0   Students compute and analyze statistical measurements for data sets:

1.1   Compute the range, mean, median, and mode of data sets.

1.2   Understand how additional data added to data sets may affect these computations of measures of central tendency.

1.3   Understand how the inclusion or exclusion of outliers affects measures of central tendency.

1.4   Know why a specific measure of central tendency (mean, median, mode) provides the most useful information in a given context.

2.0   Students use data samples of a population and describe the characteristics and limitations of the samples:

2.1   Compare different samples of a population with the data from the entire population and identify a situation in which it makes sense to use a sample.

2.2   Identify different ways of selecting a sample (e.g., convenience sampling, responses to a survey, random sampling) and which method makes a sample more representative for a population.

Algebra and Functions (Continued)

4.0 Students solve simple linear equations and inequalities over the rational numbers:

4.1 Solve two-step linear equations and inequalities in one variable over the rational numbers, interpret the solution or solutions in the context from which they arose, and verify the reasonableness of the results.

4.2 Solve multistep problems involving rate, average speed, distance, and time or a direct variation.

Measurement and Geometry

1.0 Students choose appropriate units of measure and use ratios to convert within and between measurement systems to solve problems:

1.1 Compare weights, capacities, geometric measures, times, and temperatures within and between measurement systems (e.g., miles per hour and feet per second, cubic inches to cubic centimeters).

Convert the following:

1. 80 miles/hr. = ? ft./sec.

2. 20 oz./min. = ? qts./day

1.2 Construct and read drawings and models made to scale.

1.3 Use measures expressed as rates (e.g., speed, density) and measures expressed as products (e.g., person-days) to solve problems; check the units of the solutions; and use dimensional analysis to check the reasonableness of the answer.

2.0 Students compute the perimeter, area, and volume of common geometric objects and use the results to find measures of less common objects. They know how perimeter, area, and volume are affected by changes of scale:

2.1 Use formulas routinely for finding the perimeter and area of basic two-dimensional figures and the surface area and volume of basic three-dimensional figures, including rectangles, parallelograms, trapezoids, squares, triangles, circles, prisms, and cylinders.

2.2 Estimate and compute the area of more complex or irregular two- and three-dimensional figures by breaking the figures down into more basic geometric objects.

Measurement and Geometry (Continued)

2.3   Compute the length of the perimeter, the surface area of the faces, and the volume of a three-dimensional object built from rectangular solids. Understand that when the lengths of all dimensions are multiplied by a scale factor, the surface area is multiplied by the square of the scale factor and the volume is multiplied by the cube of the scale factor.

2.4   Relate the changes in measurement with a change of scale to the units used (e.g., square inches, cubic feet) and to conversions between units (1 square foot = 144 square inches or [1 ft^2] = [144 in^2], 1 cubic inch is approximately 16.38 cubic centimeters or [1 in^3] = [16.38 cm^3]).

3.0   **Students know the Pythagorean theorem and deepen their understanding of plane and solid geometric shapes by constructing figures that meet given conditions and by identifying attributes of figures:**

3.1   Identify and construct basic elements of geometric figures (e.g., altitudes, midpoints, diagonals, angle bisectors, and perpendicular bisectors; central angles, radii, diameters, and chords of circles) by using a compass and straightedge.

3.2   Understand and use coordinate graphs to plot simple figures, determine lengths and areas related to them, and determine their image under translations and reflections.

3.3   Know and understand the Pythagorean theorem and its converse and use it to find the length of the missing side of a right triangle and the lengths of other line segments and, in some situations, empirically verify the Pythagorean theorem by direct measurement.

What is the side length of an isosceles right triangle with hypotenuse $\sqrt{72}$?

3.4   Demonstrate an understanding of conditions that indicate two geometrical figures are congruent and what congruence means about the relationships between the sides and angles of the two figures.

3.5   Construct two-dimensional patterns for three-dimensional models, such as cylinders, prisms, and cones.

Measurement and Geometry (Continued)

3.6 Identify elements of three-dimensional geometric objects (e.g., diagonals of rectangular solids) and describe how two or more objects are related in space (e.g., skew lines, the possible ways three planes might intersect).

True or false? If true, give an example. If false, explain why.

Two planes in three-dimensional space can:

1. Intersect in a line.

2. Intersect in a single point.

3. Have no intersection at all.

Statistics, Data Analysis, and Probability

1.0 Students collect, organize, and represent data sets that have one or more variables and identify relationships among variables within a data set by hand and through the use of an electronic spreadsheet software program:

1.1 Know various forms of display for data sets, including a stem-and-leaf plot or box-and-whisker plot; use the forms to display a single set of data or to compare two sets of data.

1.2 Represent two numerical variables on a scatterplot and informally describe how the data points are distributed and any apparent relationship that exists between the two variables (e.g., between time spent on homework and grade level).

1.3 Understand the meaning of, and be able to compute, the minimum, the lower quartile, the median, the upper quartile, and the maximum of a data set.

Mathematical Reasoning

1.0 Students make decisions about how to approach problems:

1.1 Analyze problems by identifying relationships, distinguishing relevant from irrelevant information, identifying missing information, sequencing and prioritizing information, and observing patterns.

1.2 Formulate and justify mathematical conjectures based on a general description of the mathematical question or problem posed.

1.3 Determine when and how to break a problem into simpler parts.