struction intended through technology? Since the students learned with technology, is it appropriate to assess knowledge with the same technology?

Again from the perspective of a teacher, look at the sources of the activities. They were selected to display the wealth of supplementary materials available to the teacher in designing a curriculum best suited for her students. The sources are various NCTM materials, journal articles, books of activities, software companies, and conference presentations by teachers. None of the sources are published texts. (Those are the sources for activities in the next chapter.) As you advance as a professional, be aware of the rich materials to find with your colleagues in settings for teacher enhancement for both your growth in mathematical understanding and to add excitement for your students' learning.

MANIPULATIVES

A manipulative is an object that students can handle and move to address mathematical concepts through visual and kinesthetic senses. According to educational psychology, some students are kinesthetic learners, so touching objects may be especially effective for them. Further, research indicates that all learners can benefit from manipulatives. Suydam and Higgins (1977), after reviewing a number of studies, concluded that elementary-level students who used manipulatives scored significantly higher in achievement. Ashlock maintained that students learn patterns of errors because they move too quickly into abstraction when they still need practice with concrete objects (Driscoll, 1981). Hunting (quoted in Suydam and Higgins, 1977) advocated the use of physical materials in teaching fractions due to his finding of a lack of understanding of equivalent fractions for fourth-, sixth-, and eighth-grade students. Manipulatives have moved from the elementary school to the middle school and are now frequently employed at the high-school level as the supporting innovative curriculum is being developed.

From Concrete to Abstract With Manipulatives

There is a long history of support for the practice of advancing students' mathematical understanding by moving from concrete experiences to abstract thinking. The abstraction of symbolic representation assists us in making deep connections in mathematics and even enables us to follow logical thinking into uncharted territories to expand the field of mathematics. However, the sophistication necessary for understanding abstract representations and the facile use of symbolic mathematics is not easily gained. This passage from concrete to abstract thinking has been described as "not so much a bridge as the entire journey" (Driscoll, 1981, p. 11). Even as late as middle school, many students find their understanding of abstraction grounded in their perceptions of the real world. Piaget's research indicated that symbolic representations have meanings for students as old as 12 only if they are closely tied to concrete representations (Driscoll, 1981).

Educational researchers Piaget, Skemp, and the van Hieles concluded that students proceed through learning stages. Manipulatives are significant aids to learning in all four of Piaget's stages, which he identifies as sensormotor, preoperational, concrete operational, and formal operational. In contrast, Skemp recognizes two levels of development: students at the second level of mental associations build on the first level, which is based in physical activities (Kennedy, 1986). In the 1950s Dutch educators Dina van Hiele-Geldof and Pierre van Hiele became concerned with secondary level students' difficulties in learning geometry. They studied how students learn geometry and identified five levels of instructional experiences through which students pass to reach the highest level of ability to establish rigorous theorems. These levels, known as the van Hiele levels, ask students to use visual, verbal, drawing, logical, and applied skills as they pass through five levels: recognition, analysis, ordering, deduction, and rigor. Students must master large portions of prior levels before attaining proficiency at advanced levels (Hoffer, 1981). There is no single route or rate to abstraction, but research has shown that manipulatives can be important to students taking this journey (Driscoll, 1981).

Using Manipulatives

The most significant influence on the successful use of manipulatives is the quality of teacher-conducted instruction in their use. The objects or representations themselves cannot teach. Just as a first-time user of the graphing calculator may not "see" a straight line for a linear equation (i.e., she sees the fine zigzag instead), a student may become distracted by manipulative color or shapes that are extraneous to the mathematical concept under study.

Further, facility with manipulatives does not guarantee understanding. Lessons with manipulatives must be carefully designed to advance students' thinking from "naive" to "expert." Thus, teachers must consider a number of issues in using manipulatives to make mathematics more accessible for students.

For example, before employing manipulatives, a teacher must avoid some assumptions. Clements and McMillen (1996) warn us of three. First, do not assume that the images you see when using manipulatives are identical to those seen by your students. You approach the activity from an expert position and can easily appreciate the connections between the concrete object(s) and the appropriate abstraction(s). In contrast, students come from a naive perspective. Their use of a specific manipulative may or may not lead them to the abstraction. Second, do not assume when students maneuver objects in specified ways that the expected mental associations automatically follow. For example, Gravemeijer, an educational researcher, found that students' patterns of thinking following manipulations on an abacus did not necessarily follow those patterns intended by the teachers. Third, realize that manipulatives can be used in a rote manner. Students may blindly follow a set of rules with manipulatives without really understanding the underlying ideas, just as they might do with symbolic algorithms. The effective use of manipulatives requires your thoughtful consideration, with manipulative use as a means—not an end—to developing students' thinking with your guidance (Clements & McMillen, 1996).

The following suggestions outline some considerations for effective and appropriate use of manipulatives:

- Select manipulatives that the *students* will use. Although this may seem an obvious suggestion, all too often teachers resort to carrying out demonstrations rather than supplying students with manipulatives due, perhaps, to a lack of time or lack of money for adequate sets of materials. The benefits of manipulatives are reduced if students cannot handle and move the objects themselves. If cost is a concern, consider using the kinds of manipulatives that can be made, laminated in plastic, and reused for many years.
- Select manipulatives that allow multiple applications. As you will see in the following examples, some manipulatives can be used for differing concepts and with students of different levels. Thus, you can use them throughout the year in various classes for students within a broad spectrum of ability. Manipulatives serving a number of purposes enable students to be creative in their applications in a variety of conceptual settings and help you stretch your materials budget. Also, when students are familiar with a manipulative, they know how to handle the objects and need less time for exploration. Further, when manipulatives can be used in a variety of ways, fewer are needed, and thus the problem of adequate and convenient storage is lessened.
- At the beginning of the year impress upon students the importance of proper treatment of all classroom materials. Even if the materials consist of nothing more than 3-by-5-inch cards, time and money were invested to procure them. They must be handled with respect.
- Give general rules for using manipulatives, including when they can be used and where they will be stored. When introducing each new manipulative discuss its safe and considerate use. Be explicit in your instructions of how to put the equipment away.
- When using a manipulative new to your students, always allow exploration time. Usually, they are intrigued and wish to experiment with the objects. If you try to go directly to a structured lesson you will lose the students. Plan for "play" time and ask students to observe some properties of the manipulatives in their initial examination.
- In introducing a topic, use one specific manipulative. After thoroughly working with the one representation, you may then allow students to try other concrete representations, especially if they suggest them. However, do not overwhelm students with a number of different representations at the early stages of development of a concept.
- Do encourage students to use manipulatives when they feel it would assist their learning. It is good practice to have manipulatives available to students upon request. Having a variety of manipulatives available allows students to select the representations most helpful to them. The practice of calling upon different manipulatives is particularly effective after the introduction of a concept, while students are extending their knowledge of the area of study.
- Remember that students advance at different rates. Some students might choose to use manipulatives long after other students prefer to move quickly to pictorial or symbolic representations. Students should not be required to use manipulatives if they can develop other methods of achieving understanding. Pictorial and symbolic representations can be less cumbersome than physical objects, especially for students with good visualization skills.
- Be certain that all students have an opportunity to manipulate the objects. If one set of materials

is given to a cooperative group, structure the activity so that all students are physically involved. This may be done by requiring a different exercise by each member of the group. To the same end, you might assure equity by incorporating an evaluation into the activity by randomly selecting one student in each group to demonstrate the group's ability either to you or to the class.

- Consider the appropriateness of the manipulative to the age of the students. Unfortunately, some excellent materials for practice with core concepts are decorated with cute pictures that may turn off high-school students. I (SM) was amazed at how much more readily my high-school students took to secondary multilink cubes colored only white, black, and gray than to the more colorful multilink cubes used at the elementary school, although the same concepts could be developed by either type. Introduce any manipulative activity by indicating the important underlying mathematics so that students perceive it is a worthy exercise, regardless of their grade level or ability level.
- Organize the manipulatives in transparent bags, appropriately sized boxes, or containers that are easy to take out and put away neatly.

Bloomer and Carlson (1993) give the following excellent advice:

> We don't allow students to say "I don't get it," but have them say, "I don't *quite* get it yet." This creates an expectation and trust in the students that they will understand the materials. They know
>
> - it will be presented in several ways, with several different manipulatives;
> - it's all right if they don't understand well at first;
> - they can use the manipulative for as long as they feel a need; and
> - they have a teacher, each other, and their previous experience to draw on for help.
>
> Their comfort and confidence levels increase, and the amount of math anxiety decreases. (p. 5)

A complaint from some teachers is that they have no time to use manipulatives because they have so much material "to cover." If you feel the pressure to "cover" the text, ask yourself, "Who is covering the material?" All too frequently it is the teacher and not the students. Since you already know the mathematics, it makes no sense to "cover" the material without instilling student understanding. Although at first it may seem that manipulatives are too time consuming, remember the Chinese adage. "I hear, and I forget. I see, and I remember. I do, and I understand." For many students, *doing* requires making concrete ties to their real-world experiences and the doing is enhanced by manipulatives. Teachers find that as concepts become more difficult, students familiar with procedures using manipulatives often get new skills as a "free ride" (Bohan, 1990). Bloomer and Carlson (1993) say:

> Time is short. Manipulative and inquiry lessons take longer, both initially and because you need to include the connecting stage. You may fear you will never cover the curriculum or that students will not have the written skills to pass the required tests. Halfway through the year, you may start to get that familiar feeling of panic. We know it happens because we've been there.
>
> An interesting thing happens. . . . Because (students) often see advanced lessons as requiring not new skills but logical extensions of what they have already learned, they begin to grasp concepts more quickly. Therefore, subsequent skill acquisition occurs at a much faster pace. (pp. 3–4)

In the next section of this chapter, common uses of some popular manipulatives are illustrated. All of the manipulatives described can be purchased, and most can be made. Certainly, the examples are by no means exhaustive of the many types of manipulatives now available. New manipulative applications developed and advanced by exemplary teachers are regularly presented in the journals of the NCTM. Further, at conferences you will also find many great ideas for using manipulatives. With experience you will invent manipulatives and explore new ways of using them. The following section simply offers a beginning.

Set 3.1 Discussion Questions

Questions with an asterisk appear in the Message Board section of the Companion Website at ***http://www.prenhall.com/huetinck.*** Go to Chapter 3 and click on the Message Board to find and respond to the question.

*1. What teacher behaviors maximize the effective use of manipulatives in the classroom?
2. Is it possible for students to become too dependent upon manipulatives? Explain.

Fraction Stacks, Pattern Blocks, Fraction Bars, and Cuisenaire Rods

Fraction stacks, pattern blocks, fraction bars, fraction circles, and Cuisenaire rods are different manipulatives helpful in instruction with fractions. (These manipulatives also are effective in other mathematical contexts. For example, pattern blocks