MATH 100 **Spring 2008**

Solutions to Practice Problems for Final Exam

1. Mr. Anderson had \$600 after he gave $\frac{2}{5}$ of his money to his wife. So \$600 was $\frac{3}{5}$ of the original amount. Therefore the original amount was \$1,000.

2. The green portion is $\frac{3}{3+4+2} = \frac{3}{9} = \frac{1}{3}$ of the whole pole. The black portion is $\frac{2}{3+4+2} = \frac{2}{9}$ of the whole pole. Therefore 30 cm of the pole is painted green and 20 cm is painted black.

3. The first half of the trip was $2 \cdot 75 = 150$ km. Thus the whole trip was 300 km. Since the average speed for the whole trip was 60 km/h, it took 5 hours. Then the second half of the trip took 3 hours. Therefore the average speed for second half of the trip was $150/3 = 50$ km/h.

4. Let the side of each square be 1 unit. Then the diagonal of a square is $\sqrt{2}$ units long. Thus the perimeter is $20\sqrt{2} + 20$ units. The area is 60 square units.

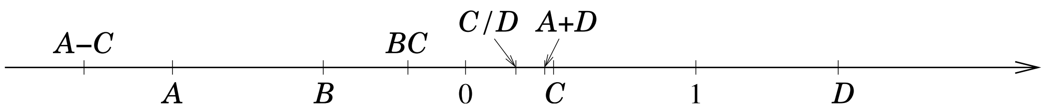
5. (a) True ($24 = 8 \cdot 3$)
   (b) False (15 is not 30 times an integer)
   (c) True ($70 = 7 \cdot 10$)
   (d) False (10 is not 30 times an integer)
   (e) False (1 is neither prime nor composite)
   (f) False (for example, 1 and 2 are integers, but $\frac{1}{2}$ is not)
   (g) True (by definition of real numbers)
   (h) True (there are infinitely many prime numbers, but there are only finitely many numbers not exceeding 1,000,000)
   (i) False ($\pi \approx 3.14$, but not exactly equal)
   (j) False ($3.14 = \frac{314}{100}$ is a quotient of integers, thus rational by definition)

6. $10^{15} = 2^{15} \cdot 5^{15}$, $15^{10} = 3^{10} \cdot 5^{10}$;
   $GCF(10^{15}, 15^{10}) = 5^{10}$, $LCM(10^{15}, 15^{10}) = 2^{15} \cdot 3^{10} \cdot 5^{15}$

7. In order for the average of six scores to be 90, the sum of these six scores must be $90 \cdot 6 = 540$. Therefore Cathy needs to get $540 - (91 + 80 + 96 + 91 + 83) = 540 - 441 = 99$ on the sixth exam.

8. $1+2+3+\ldots+2008 = \dfrac{(1+2+3+\ldots+2008) + (2008+2007+2006+\ldots+1)}{2} = \dfrac{2009+2009+2009+\ldots+2009}{2} = \dfrac{2009 \cdot 2008}{2} = 2009 \cdot 1004 = 2017036.$

9. Approximate locations of $A + D$, $A - C$, BC, and C/D are shown below:



10. Long division gives $\frac{1}{7} = 0.142857142857\ldots$. We see that the six digits 142857 repeat, so the first 24 digits will consist of 4 of these 6-tuples, and the 25th digit will be 1.

11. $(2x^2 + 3x + 1)(x^2 + 4x + 5) = 0$
$2x^2 + 3x + 1 = 0$ or $x^2 + 4x + 5 = 0$

$x = \frac{-3 \pm \sqrt{9-8}}{4}$ or $x = \frac{-4 \pm \sqrt{16-20}}{2}$

$x = \frac{-3 \pm 1}{4}$ or $x = \frac{-4 \pm \sqrt{-4}}{2}$

$x = \frac{-2}{4}$ or $x = \frac{-4}{4}$ or $x = \frac{-4 \pm 2i}{2}$

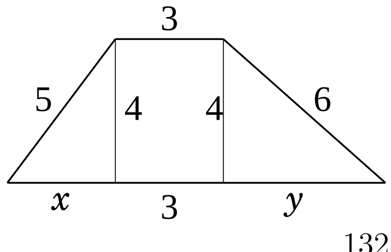
$x = \frac{-1}{2}$ or $x = -1$ or $x = -2 + i$ or $x = -2 - i$

Thus the answers are as follows:

(a) there are no natural solutions
(b) one integer solution: $x = -1$
(c) two real solutions: $x = \frac{-1}{2}$, $x = -1$
(d) four complex solutions: $x = \frac{-1}{2}$, $x = -1$, $x = -2 + i$, $x = -2 - i$

12. The correct answer is (e): only a parallelogram with diagonals that are perpendicular to each other (in which case it is a rhombus) can be made.
All angles of an equilateral triangle are $60°$, so it cannot not have a right angle. Two sides of a trapezoid are parallel, so if one angle is right, there must be another right angle. An obtuse triangle always has two acute angles (because the sum of all angles must add up to $180°$). All angles of a regular pentagon are $108°$ (a pentagon can be divided into 3 triangles, the sum of all angles of each triangle is $180°$, so all the angles of a pentagon add up to 540).

13. Using the Pythagorean theorem, we find that the lengths x and y are: $x = \sqrt{5^2 - 4^2} = \sqrt{9} = 3$ and $y = \sqrt{6^2 - 4^2} = \sqrt{20} = 2\sqrt{5}$. Thus the base is $3 + 3 + 2\sqrt{5} = 6 + 2\sqrt{5}$. The perimeter is $5 + 3 + 6 + 6 + 2\sqrt{5} = 20 + 2\sqrt{5}$. The area is $\dfrac{(3 + 6 + 2\sqrt{5}) \cdot 4}{2} = 18 + 4\sqrt{5}$.



14. The cube has 6 faces, so the area of each face is $\dfrac{132}{6} = 22$ cm^2. Thus each edge is $\sqrt{22}$ cm long. The volume is $(\sqrt{22})^3 = 22\sqrt{22}$ cm^3.