MATH 100 **Spring 2008**

Solutions to Practice Problems for Test 1

1. Possible values of coins are 1, 5, 10, 25 cents. There must be at least two pennies because all other values are divisible by 5 and the amount ends in 7. The total value of the other five coins is 55 cents. Since five times 10 is only 50, there must be at least one quarter. Then the total value of the remaining four coins is 30 cents. The only possible combination is two nickels and two dimes. Thus I have two pennies, two nickels, two dimes, and one quarter.

2. Since Frankie is now on page 60, it must be day 7 (he has read 54 pages in 6 days and will have read 63 pages by the end of the 7th day). Johnny has read 66 pages in 6 days, and will have read 77 pages by the end of the 7th day. So Johnny must be somewhere between 66th and 77th pages. However, we cannot tell what page he is on right now, since we don't know how many pages he has already read today.

3. Let x be the amount that the youngest child received. Then the eldest child received $3x$, and the middle child received $x + 14,000$. The total amount is $x + 3x + x + 14,000 = 64,000$. Simplifying, we get $5x + 14,000 = 64,000$. Subtracting 14,000 from both sides gives $5x = 50,000$. Therefore $x = 10,000$. Thus the youngest child received \$10,000, the eldest child received \$30,000, and the middle child received \$24,000.

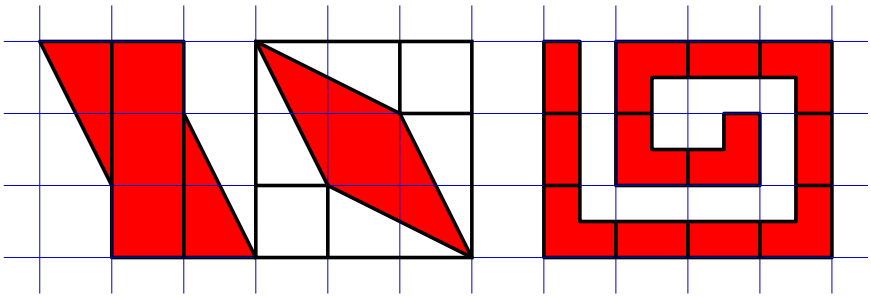
4. Next time Bill and Sue will be off again in the number of days that is the smallest common multiple of 6 and 8, i.e. 24. So it will be 23 nights before they will be off again.

5. Note that $4^{10} = (2^2)^{10} = 2^{20}$. Thus all factors of this number are powers of two: 1, 2, 2^2, 2^3, ..., 2^{19}, 2^{20}. There are 21 positive factors total.

6. The price of the stuffed cat must be a factor of both 95 and 115. Since $95 = 5 \cdot 19$, $115 = 5 \cdot 23$, and 19 and 23 are prime, the only common factors are 5 and 1. We are told that the price is greater than \$1. So it is \$5. Therefore 19 toy cats were sold on Monday and 23 cats were sold on Tuesday.

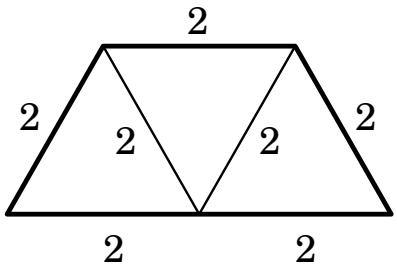
7. Let's divide the first figure into three pieces: one 3×1 rectangle and two right triangles (see picture below). The area of the rectangle is 3 square units and the area of each triangle is 1 square unit (notice that each of these triangles is half of a 2×1 rectangle). Thus the total area of the figure is $3 + 1 + 1 = 5$ square units.
   The second figure can be inscribed in a 3×3 square whose area is 9. The part outside the figure can be divided into six pieces: two unit squares and four right triangles. The area of each of these triangles is 1 (as in the previous figure). Thus the total area of the outside part is $2 \cdot 1 + 4 \cdot 1 = 6$ square units. Then the area of the figure is $9 - 6 = 3$ square units.
   The third figure can be divided into 12 pieces: six $1 \times \frac{1}{2}$ rectangles and 6 L-shaped pieces. The area of each $1 \times \frac{1}{2}$ rectangle is $\frac{1}{2}$, and the area of each L-shaped piece

is $\frac{3}{4}$. Thus the total area of the figure is $6 \cdot \frac{1}{2} + 6 \cdot \frac{3}{4} = 3 + \frac{9}{2} = 7\frac{1}{2}$.



8. When each side of a figure is made 10 times longer, the area becomes $10^2 = 100$ times larger. Thus the area of an equilateral triangle with side 10 is approximately $0.433 \cdot 100 = 43.3$ square units.

9. A trapezoid with sides 2, 2, 2, 4 can be divided into three equilateral triangles with side 2. The area of each of these triangles is approximately $0.433 \cdot 4 = 1.732$ square units (see previous problem). Then the area of the trapezoid is approximately $1.732 \cdot 3 = 5.196$ square units.



10. We know that the area of a rectangle is the product of its length and width. Since 37 is a prime number, its only factors are 37 and 1. Thus the rectangle has dimensions 37 units $\times$ 1 unit. Its perimeter is $37 \cdot 2 + 1 \cdot 2 = 74 + 2 = 76$.

11. Solution 1: In one problem discussed in class we established that $1 + 3 + 5 + \ldots + (2n - 1) = n^2$ (e.g. $1 = 1^1$, $1 + 3 = 2^2$, $1 + 3 + 5 = 3^2$, and so on). Using this pattern, we find that $1 + 3 + 5 + \ldots + 999 = 500^2 = 250,000$.
Solution 2: Rearrange the terms in the sum as follows: $1 + 3 + 5 + \ldots + 999 = (1 + 999) + (3 + 997) + (5 + 995) + \ldots + (499 + 501) = 1,000 + 1,000 + 1,000 + \ldots + 1,000 = 250 \cdot 1,000 = 250,000$.

12. $\frac{2}{3} + 1\frac{5}{6} = \frac{2}{3} + \frac{11}{6} = \frac{4}{6} + \frac{11}{6} = \frac{15}{6} = \frac{5}{3}$
$\frac{2}{3} \times 1\frac{5}{6} = \frac{2}{3} \times \frac{11}{6} = \frac{22}{18} = \frac{11}{9}$
$\frac{2}{3} \div 1\frac{5}{6} = \frac{2}{3} \div \frac{11}{6} = \frac{2}{3} \times \frac{6}{11} = \frac{12}{33} = \frac{4}{11}$

13. Write your own story problems. You may use some ideas given in Liping Ma's article on fraction division and multiplication.

14. Read the article about manipulatives.