MATH 100 **Spring 2008**

Solutions to Practice Problems for Test 2

1. Correct answer is (a): if 35 percent of all test takers scored better than Angelica, but 40 percent of all females who took the test scored better than she did, females scored a bit higher than males.

2. Correct answer is (c): a 3rd grader who answered 80% of the questions correctly did better than Cristina, thus also exceeded the minimum state proficiency cut-point in mathematics in her state.

3. Let's first order the data: 2, 3, 4, 6, 7, 8, 8, 8, 9, 9. The mean (average) is the sum of all the scores divided by the number of scores, i.e. $\frac{2+3+4+6+7+8+8+8+9+9}{10} = \frac{64}{10} = 6.4$. The median is the average of two scores in the middle, i.e. $\frac{7+8}{2} = \frac{15}{2} = 7.5$. The mode is the score that occurs most often, i.e. 8.

4. (a) True. Two rational numbers can be written as $\frac{a}{b}$ and $\frac{c}{d}$ where a, b, c, d are integers and $b \neq 0$, $d \neq 0$. Their product is $\frac{ac}{bd}$. Since ac and bd are integers and $bd \neq 0$, the product is a rational number.

   (b) False. For example, 4 is a natural number, but $\sqrt{4} = 2$ is not irrational: $2 = \frac{2}{1}$.

   (c) False. Both $\sqrt{2}$ and $-\sqrt{2}$ are irrational, but their sum is $\sqrt{2} + (-\sqrt{2}) = 0$ is not irrational: $0 = \frac{0}{1}$.

   (d) False. Prime numbers are integers, and all integers are rational.

   (e) True. If r is an integer, it can be written as $\frac{r}{1}$, thus it is rational.

   (f) Ture. This follows from the definition: a number is called rational if it can be written as a quotient of integers, and it is called irrational otherwise.

   (g) False. Periodic (repeating) infinite decimal numbers are rational.

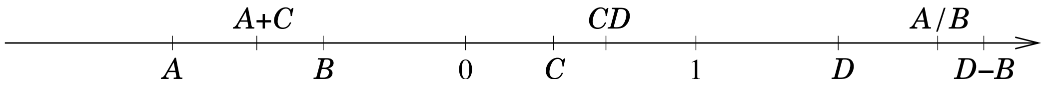
   (h) True. We will argue that it is not possible for such a sum to be rational. Suppose r is a rational number and x is an irrational number such that $r + x$ is rational. Then we can write the rational numbers as quotients of integers: $r = \frac{a}{b}$ and $r + x = \frac{c}{d}$. Then $x = (r + x) - r = \frac{c}{d} - \frac{a}{b} = \frac{cb-ad}{db}$. Since $cb - ad$ and db are integers, x is rational. However, we assumed that it was irrational. This is impossible.

   (i) False. For example, 0 is rational and $\sqrt{2}$ is irrational, but their product is not irrational: $0 \cdot \sqrt{2} = 0$.

5. For example, 3.4565 is between 3.456 and 3.457 and is rational: $3.4565 = \frac{34565}{10000}$.

6. We know that $\sqrt{2} = 1.4142\ldots$ is irrational. By adding the rational number 3.8 to it, we obtain the irrational number $\sqrt{2} + 3.8 = 5.2142\ldots$ which is between 5.2 and 5.3.

7. (a) True, since $18 = 6 \cdot 3$ and 3 is an integer.
   (b) False, since 3 is not 9 times an integer.
   (c) True, since $7 = 7 \cdot 1$.
   (d) False, since 4 is not 8 times an integer.
   (e) True, since $20 = 5 \cdot 4$.
   (f) True, since $10 = 10 \cdot 1$.
   (g) True, since $12 = 3 \cdot 4$.
   (h) False, since 8 is not 24 times an integer.
   (i) True, since $13 = 13 \cdot 1$.
   (j) False, since 5 is not 30 times an integer.
   (k) True, since $30 = 2 \cdot 15$.
   (l) True, since $100 = 100 \cdot 1$.
8. $300 = 2 \cdot 2 \cdot 3 \cdot 5 \cdot 5$.
9. First we'll find the prime factorization of 630: $630 = 2 \cdot 3 \cdot 3 \cdot 5 \cdot 7$. We see that 300 and 630 share three factors: 2, 3, and 5. Thus the greatest commond factor of 300 and 630 is $2 \cdot 3 \cdot 5 = 30$ and the least common multiple is $2 \cdot 2 \cdot 3 \cdot 3 \cdot 5 \cdot 5 \cdot 7 = 6300$.
10. Answers are shown below:



11. First we will find all real solutions. The product of real numbers is 0 if and only if at least one of the factors is 0, thus we have $x - 2 = 0$ or $x + 2 = 0$ or $2x + 1 = 0$. We get three real roots: $x = 2$, $x = -2$, and $x = -0.5$. Therefore the answers are as follows:
    (a) 2
    (b) 2 and -2
    (c) 2, -2, and -0.5
12. Again, first we will find all real solutions. Multiplying both sides of the equation by $(x+2)(x+3)$ we get $3(x+3) = 2(x+2)$. Multiplying out gives $3x+9 = 2x+4$ which simplifies to $x + 5 = 0$. Thus there is only one real root: $x = -5$. Therefore the answers are as follows:
    (a) no solutions
    (b) -5
    (c) -5