


Kindergarten Mathematics Content Standards

By the end of kindergarten, students understand small numbers, quantities, and simple shapes in their everyday environment. They count, compare, describe and sort objects, and develop a sense of properties and patterns.

Note: The sample problems illustrate the standards and are written to help clarify them. Some problems are written in a form that can be used directly with students; others will need to be modified, particularly in the primary grades, before they are used with students.

The symbol  identifies the key standards for kindergarten.

Number Sense

1.0 Students understand the relationship between numbers and quantities (i.e., that a set of objects has the same number of objects in different situations regardless of its position or arrangement):

1.1 Compare two or more sets of objects (up to 10 objects in each group) and identify which set is equal to, more than, or less than the other.

Are there more circles or more triangles in the following collection?



1.2 Count, recognize, represent, name, and order a number of objects (up to 30).

Which numbers are missing?












11, 12, 13, , , 16, 17, , , , 21, 22, 23, 24.

1.3 Know that the larger numbers describe sets with more objects in them than the smaller numbers have.

2.0 Students understand and describe simple additions and subtractions:

2.1 Use concrete objects to determine the answers to addition and subtraction problems (for two numbers that are each less than 10).

Pair up as many groups of beans from the left column with groups of beans from the right column so that each group adds up to 10 beans.

Number Sense (Continued)

Chapter 2
Mathematics
Content
Standards

3.0 Students use estimation strategies in computation and problem solving that involve numbers that use the ones and tens places:

- 3.1 Recognize when an estimate is reasonable.

Algebra and Functions

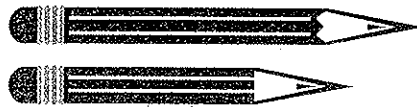
Kindergarten

1.0 Students sort and classify objects:

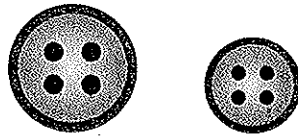
- 1.1** Identify, sort, and classify objects by attribute and identify objects that do not belong to a particular group (e.g., all these balls are green, those are red).

Students compare objects:

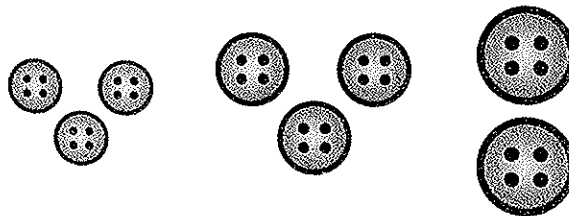
1. Which pencil is longer? Shorter?



2. Describe how the following 2 objects are the same or different.




3. Show students buttons sorted into 3 sets as shown and ask them to identify how buttons were sorted.





Grade One Mathematics Content Standards

Note: The sample problems illustrate the standards and are written to help clarify them. Some problems are written in a form that can be used directly with students; others will need to be modified, particularly in the primary grades, before they are used with students.

The symbol  identifies the key standards for grade one.

By the end of grade one, students understand and use the concept of ones and tens in the place value number system. Students add and subtract small numbers with ease. They measure with simple units and locate objects in space. They describe data and analyze and solve simple problems.

Number Sense**1.0 Students understand and use numbers up to 100:**

-  **1.1** Count, read, and write whole numbers to 100.
-  **1.2** Compare and order whole numbers to 100 by using the symbols for less than, equal to, or greater than ($<$, $=$, $>$).

Which of the following are correct and which are incorrect?

- (a) $75 > 76$ (b) $48 < 42$ (c) $89 > 91$
 (d) $59 < 67$ (e) $34 = 33$

- 1.3 Represent equivalent forms of the same number through the use of physical models, diagrams, and number expressions (to 20) (e.g., 8 may be represented as $4 + 4$, $5 + 3$, $2 + 2 + 2 + 2$, $10 - 2$, $11 - 3$).
- 1.4 Count and group object in ones and tens (e.g., three groups of 10 and 4 equals 34, or $30 + 4$).

A certain brand of chewing gum has 10 pieces in each pack. If there are 14 students, what is the smallest number of packs we must buy to make sure each student gets at least one piece of gum? If there are 19 students? What about 21 students?

There are 5 quarters, 9 dimes, 3 nickels, and 8 pennies. They are supposed to be put in piles of ten (coins). How many such piles can be formed by all these coins, and how many are left over?

- 1.5 Identify and know the value of coins and show different combinations of coins that equal the same value.

I have some pennies, nickels, and dimes in my pocket. I reach in and pull out three coins. How much money might I have? List all the possibilities.

Number Sense (Continued)**2.0 Students demonstrate the meaning of addition and subtraction and use these operations to solve problems:**

- 2.1** Know the addition facts (sums to 20) and the corresponding subtraction facts and commit them to memory.

I had 10 cupcakes, but I ate 3 of them. How many cupcakes do I have left?
How many if I had 18 and ate 5?

- 2.2** Use the inverse relationship between addition and subtraction to solve problems.

- 2.3** Identify one more than, one less than, 10 more than, and 10 less than a given number.

- 2.4** Count by 2s, 5s, and 10s to 100.

Which numbers are missing?

24, 26, 28, 30, __, __, 36, __, 40, 42, 44, __, __, 50

15, 20, 25, 30, __, __, 45, __, 55, 60, __, 70, __, 80

- 2.5** Show the meaning of addition (putting together, increasing) and subtraction (taking away, comparing, finding the difference).

- 2.6 Solve addition and subtraction problems with one- and two-digit numbers (e.g., $5 + 58 = \underline{\quad}$).

Figure out how many pages I have read so far this week if I read 16 pages on Monday, 9 pages on Tuesday, none on Wednesday, and 7 pages on Thursday.

- 2.7 Find the sum of three one-digit numbers.

3.0 Students use estimation strategies in computation and problem solving that involve numbers that use the ones, tens, and hundreds places:

- 3.1 Make reasonable estimates when comparing larger or smaller numbers.

Algebra and Functions

1.0 Students use number sentences with operational symbols and expressions to solve problems:

- 1.1 Write and solve number sentences from problem situations that express relationships involving addition and subtraction.

Do the following problems in succession:

1. Marie had some pencils in her desk. She put 5 more in her desk. Then she had 14. How many pencils did she have in her desk to start with?
 2. Eddie had 14 helium balloons. A number of them floated away. He had 5 left. How many did he lose?
 3. Nina had 14 seashells. That was 5 more than Pedro had. How many seashells did Pedro have?
 4. $5 + () = 6$? $() + 12 = 14$?
- 1.2 Understand the meaning of the symbols +, −, =.
- 1.3 Create problem situations that might lead to given number sentences involving addition and subtraction.

Measurement and Geometry

1.0 Students use direct comparison and nonstandard units to describe the measurements of objects:

- 1.1 Compare the length, weight, and volume of two or more objects by using direct comparison or a nonstandard unit.

Measure your desk by using the length of a ballpoint pen. How many ballpoint pens would be roughly equal to the length of your desk? The width of your desk? Which is longer?

- 1.2 Tell time to the nearest half hour and relate time to events (e.g., before/after, shorter/longer).


2.0 Students identify common geometric figures, classify them by common attributes, and describe their relative position or their location in space:

- 2.1 Identify, describe, and compare triangles, rectangles, squares, and circles, including the faces of three-dimensional objects.

Describe the shape of a page in your textbook and compare it to the face of the clock on the wall.

Grade Two Mathematics Content Standards


Note: The sample problems illustrate the standards and are written to help clarify them. Some problems are written in a form that can be used directly with students; others will need to be modified, particularly in the primary grades, before they are used with students.

The symbol  identifies the key standards for grade two.

By the end of grade two, students understand place value and number relationships in addition and subtraction, and they use simple concepts of multiplication. They measure quantities with appropriate units. They classify shapes and see relationships among them by paying attention to their geometric attributes. They collect and analyze data and verify the answers.


Number Sense

1.0 Students understand the relationship between numbers, quantities, and place value in whole numbers up to 1,000:


 1.1 Count, read, and write whole numbers to 1,000 and identify the place value for each digit.


1.2 Use words, models, and expanded forms (e.g., $45 = 4 \text{ tens} + 5$) to represent numbers (to 1,000).

Kelly has 308 stickers. How many sets of hundreds, tens, and ones does she have?

 1.3 Order and compare whole numbers to 1,000 by using the symbols $<$, $=$, $>$.

2.0 Students estimate, calculate, and solve problems involving addition and subtraction of two- and three-digit numbers:

 2.1 Understand and use the inverse relationship between addition and subtraction (e.g., an opposite number sentence for $8 + 6 = 14$ is $14 - 6 = 8$) to solve problems and check solutions.

 2.2 Find the sum or difference of two whole numbers up to three digits long.

Use drawings of tens and ones to help find the sum $37 + 17$ and the difference $25 - 19$. Now do the same problems again using addition and subtraction algorithms.

2.3 Use mental arithmetic to find the sum or difference of two two-digit numbers.

In a game, Mysong and Naoki are making addition problems. They make two 2-digit numbers out of the four given numbers 1, 2, 3, and 4. Each number is used exactly once. The winner is the one who makes two numbers whose sum is the largest. Mysong had 43 and 21, while Naoki had 31 and 24. Who won the game? How do you know? Show how you can beat both Mysong and Naoki by making up two numbers with a larger sum than either (Adapted from TIMSS, gr. 4, V-4). (This problem also supports Mathematical Reasoning Standard 1.0.)

Number Sense (Continued)**3.0 Students model and solve simple problems involving multiplication and division:**

- 3.1** Use repeated addition, arrays, and counting by multiples to do multiplication.

Draw a simple picture of seating 30 people in rows of 10. Show and explain how this is related to multiplication. Do this also for rows of 3, and again for rows of 5.

- 3.2** Use repeated subtraction, equal sharing, and forming equal groups with remainders to do division.

Kim decides to store away his marbles. He knows there are bags that hold up to 10 marbles in each. Kim has 38 marbles, and he tries to spend money on as few bags as he can. How many bags does he have to buy? How many if he has 51 marbles? (Keep in mind that there is no such thing as “half a bag” or “part of a bag.”)

- 3.3** Know the multiplication tables of 2s, 5s, and 10s (to “times 10”) and commit them to memory.

4.0 Students understand that fractions and decimals may refer to parts of a set and parts of a whole:

- 4.1** Recognize, name, and compare unit fractions from $\frac{1}{12}$ to $\frac{1}{2}$.

True or false?

- One-fourth of a pie is larger than one-sixth of the same pie.
- $\frac{1}{4} > \frac{1}{3}$
- $\frac{1}{7} < \frac{1}{9}$

- 4.2** Recognize fractions of a whole and parts of a group (e.g., one-fourth of a pie, two-thirds of 15 balls).

- 4.3** Know that when all fractional parts are included, such as four-fourths, the result is equal to the whole and to one.

5.0 Students model and solve problems by representing, adding, and subtracting amounts of money:

- 5.1** Solve problems using combinations of coins and bills.

Lee has a wallet with 5 nickels, 9 dimes, and dollar bills. In how many ways can he pay with correct change for a pen worth \$1.15? What about one worth 65 cents?

Number Sense (Continued)

- 5.2** Know and use the decimal notation and the dollar and cent symbols for money.

Which of the following show a correct use of symbols for money?

1. ¢32
2. 72¢
3. \$1.25
4. 2.57\$

- 6.0** Students use estimation strategies in computation and problem solving that involve numbers that use the ones, tens, hundreds, and thousands places:

- 6.1 Recognize when an estimate is reasonable in measurements (e.g., closest inch).

Algebra and Functions

- 1.0** Students model, represent, and interpret number relationships to create and solve problems involving addition and subtraction:

- 1.1** Use the commutative and associative rules to simplify mental calculations and to check results.

Draw pictures using dots to show:

1. Why $11 + 18 = 18 + 11$
2. Why $(11 + 5) + 17 = 11 + (5 + 17)$

If you know that $379 + 363 = 742$, what is the sum of $363 + 379$?

- 1.2 Relate problem situations to number sentences involving addition and subtraction.


Three classes at your school will see a play together in a large room. Room 1 has 18 students, Room 2 has 34 students, and Room 3 has 19 students. Figure out how many seats you will need. If Room 2 drops out but Room 4 with 29 students joins in, how many seats will you need then?

- 1.3 Solve addition and subtraction problems by using data from simple charts, picture graphs, and number sentences.

Grade Three Mathematics Content Standards

By the end of grade three, students deepen their understanding of place value and their understanding of and skill with addition, subtraction, multiplication, and division of whole numbers. Students estimate, measure, and describe objects in space. They use patterns to help solve problems. They represent number relationships and conduct simple probability experiments.

Note: The sample problems illustrate the standards and are written to help clarify them. Some problems are written in a form that can be used directly with students; others will need to be modified, particularly in the primary grades, before they are used with students.

The symbol  identifies the key standards for grade three.


Number Sense

1.0 Students understand the place value of whole numbers:


1.1 Count, read, and write whole numbers to 10,000.

What is the smallest whole number you can make using the digits 4, 3, 9, and 1? Use each digit exactly once (Adapted from TIMSS gr. 4, T-2).

1.2 Compare and order whole numbers to 10,000.

 1.3 Identify the place value for each digit in numbers to 10,000.


1.4 Round off numbers to 10,000 to the nearest ten, hundred, and thousand.

 1.5 Use expanded notation to represent numbers (e.g., $3,206 = 3,000 + 200 + 6$).

True or false?

$$3,102 \times 3 = 9,000 + 300 + 6$$

2.0 Students calculate and solve problems involving addition, subtraction, multiplication, and division:

 2.1 Find the sum or difference of two whole numbers between 0 and 10,000.

1. $591 + 87 = ?$

2. $1,283 + 6,074 = ?$

3. $3,215 - 2,876 = ?$

To prepare for recycling on Monday, Michael collected all the bottles in the house. He found 5 dark green ones, 8 clear ones with liquid still in them, 11 brown ones that used to hold root beer, 2 still with the cap on from his parents' cooking needs, and 4 more that were oversized. How many bottles did Michael collect? (This problem also supports Mathematical Reasoning Standard 1.1.)

Number Sense (Continued)

- 2.2** Memorize to automaticity the multiplication table for numbers between 1 and 10.
- 2.3** Use the inverse relationship of multiplication and division to compute and check results.
- 2.4** Solve simple problems involving multiplication of multidigit numbers by one-digit numbers ($3,671 \times 3 = \underline{\quad}$).
- 2.5 Solve division problems in which a multidigit number is evenly divided by a one-digit number ($135 \div 5 = \underline{\quad}$).
- 2.6 Understand the special properties of 0 and 1 in multiplication and division.
- True or false?
1. $24 \times 0 = 24$
 2. $19 \div 1 = 19$
 3. $63 \times 1 = 63$
 4. $0 \div 0 = 1$
- 2.7 Determine the unit cost when given the total cost and number of units.
- 2.8 Solve problems that require two or more of the skills mentioned above.
- A price list in a store states: pen sets, \$3; magnets, \$4; sticker sets, \$6.
How much would it cost to buy 5 pen sets, 7 magnets, and 8 sticker sets?
- A tree was planted 54 years before 1961. How old is the tree in 1998?
- A class of 73 students go on a field trip. The school hires vans, each of which can seat a maximum of 10 students. The school policy is to seat as many students as possible in a van before using the next one. How many vans are needed?

Number Sense (Continued)

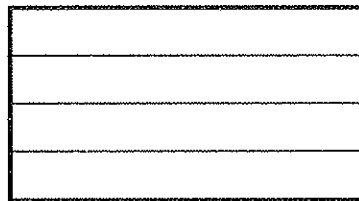
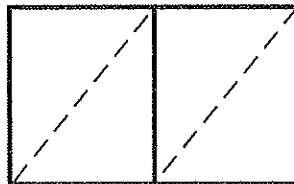
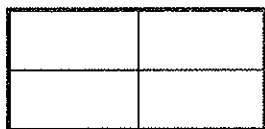
3.0 Students understand the relationship between whole numbers, simple fractions, and decimals:

- 3.1 Compare fractions represented by drawings or concrete materials to show equivalency and to add and subtract simple fractions in context (e.g., $\frac{1}{2}$ of a pizza is the same amount as $\frac{2}{4}$ of another pizza that is the same size; show that $\frac{3}{8}$ is larger than $\frac{1}{4}$).

Grade Three

Which is longer: $\frac{1}{3}$ of a foot or 5 inches? $\frac{2}{3}$ of a foot or 9 inches?

Which rectangle is NOT divided into four equal parts? (Adapted from TIMSS gr. 4, K-8)



- 3.2 Add and subtract simple fractions (e.g., determine that $\frac{1}{8} + \frac{3}{8}$ is the same as $\frac{1}{2}$).

Find the values:

1. $\frac{1}{6} + \frac{2}{6} = ?$

2. $\frac{7}{8} - \frac{3}{8} = ?$

Number Sense (Continued)

- 3.3** Solve problems involving addition, subtraction, multiplication, and division of money amounts in decimal notation and multiply and divide money amounts in decimal notation by using whole-number multipliers and divisors.

Pedro bought 5 pens, 2 erasers and 2 boxes of crayons. The pens cost 65 cents each, the erasers 25 cents each, and a box of crayons \$1.10. The prices include tax, and Pedro paid with a ten-dollar bill. How much change did he get back?

- 3.4 Know and understand that fractions and decimals are two different representations of the same concept (e.g., 50 cents is $\frac{1}{2}$ of a dollar, 75 cents is $\frac{3}{4}$ of a dollar).

Algebra and Functions

- 1.0 Students select appropriate symbols, operations, and properties to represent, describe, simplify, and solve simple number relationships:**

- 1.1** Represent relationships of quantities in the form of mathematical expressions, equations, or inequalities.

- 1.2 Solve problems involving numeric equations or inequalities.

- 1.3 Select appropriate operational and relational symbols to make an expression true (e.g., if $4 _ 3 = 12$, what operational symbol goes in the blank?).

- 1.4 Express simple unit conversions in symbolic form (e.g., $_ \text{ inches} = _ \text{ feet} \times 12$).

If number of feet = number of yards $\times 3$, and number of inches = number of feet $\times 12$, how many inches are there in 4 yards?

- 1.5 Recognize and use the commutative and associative properties of multiplication (e.g., if $5 \times 7 = 35$, then what is 7×5 ? and if $5 \times 7 \times 3 = 105$, then what is $7 \times 3 \times 5$?).

- 2.0 Students represent simple functional relationships:**

- 2.1** Solve simple problems involving a functional relationship between two quantities (e.g., find the total cost of multiple items given the cost per unit).

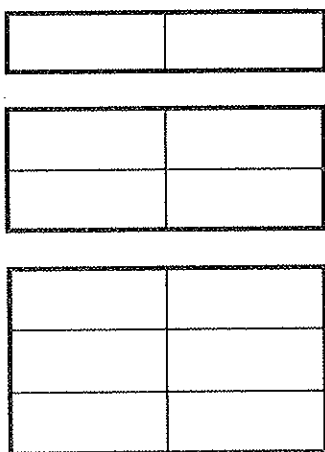
John wants to buy a dozen pencils. One store offers pencils at 6 for \$1. Another offers them at 4 for 65 cents. Yet another sells pencils at 15 cents each. Where should John purchase his pencils in order to save the most money?

Algebra and Functions (Continued)

- 2.2 Extend and recognize a linear pattern by its rules (e.g., the number of legs on a given number of horses may be calculated by counting by 4s or by multiplying the number of horses by 4).

Here is the beginning of a pattern of tiles. Assuming that the pattern continues linearly, how many tiles will be in the sixth figure?

(Adapted from TIMSS gr. 4, K-6)




Grade Three

Measurement and Geometry

- 1.0 Students choose and use appropriate units and measurement tools to quantify the properties of objects:
- 1.1 Choose the appropriate tools and units (metric and U.S.) and estimate and measure the length, liquid volume, and weight/mass of given objects.
 - 1.2 Estimate or determine the area and volume of solid figures by covering them with squares or by counting the number of cubes that would fill them.
 - 1.3 Find the perimeter of a polygon with integer sides.
 - 1.4 Carry out simple unit conversions within a system of measurement (e.g., centimeters and meters, hours and minutes).
- 2.0 Students describe and compare the attributes of plane and solid geometric figures and use their understanding to show relationships and solve problems:
- 2.1 Identify, describe, and classify polygons (including pentagons, hexagons, and octagons).

By the end of grade four, students understand large numbers and addition, subtraction, multiplication, and division of whole numbers. They describe and compare simple fractions and decimals. They understand the properties of, and the relationships between, plane geometric figures. They collect, represent, and analyze data to answer questions.





Note: The sample problems illustrate the standards and are written to help clarify them. Some problems are written in a form that can be used directly with students; others will need to be modified, particularly in the primary grades, before they are used with students.

The symbol  identifies the key standards for grade four.

Number Sense

1.0 Students understand the place value of whole numbers and decimals to two decimal places and how whole numbers and decimals relate to simple fractions.

Students use the concepts of negative numbers:

-  Read and write whole numbers in the millions.
-  Order and compare whole numbers and decimals to two decimal places.
-  Round whole numbers through the millions to the nearest ten, hundred, thousand, ten thousand, or hundred thousand.
-  Decide when a rounded solution is called for and explain why such a solution may be appropriate.

Solve each of the following problems and observe the different roles played by the number 37 in each situation:

1. Four children shared 37 dollars equally. How much did each get?
2. Four children shared 37 pennies as equally as possible. How many pennies did each get?
3. Cars need to be rented for 37 children going on a field trip. Each car can take 12 children in addition to the driver. How many cars must be rented?

1.5 Explain different interpretations of fractions, for example, parts of a whole, parts of a set, and division of whole numbers by whole numbers; explain equivalents of fractions (see Standard 4.0).

True or false?

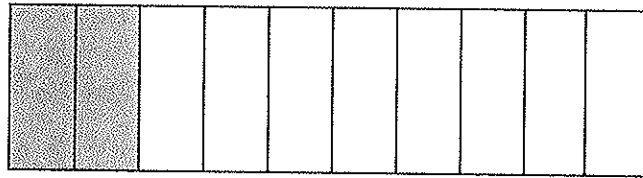
1. $1/4 > 2.54$
2. $5/2 < 2.6$
3. $12/18 = 2/3$ (Note the equivalence of fractions.)
4. $4/5 < 13/15$

Number Sense (Continued)

- 1.6 Write tenths and hundredths in decimal and fraction notations and know the fraction and decimal equivalents for halves and fourths (e.g., $\frac{1}{2} = 0.5$ or 0.50 ; $\frac{3}{4} = 1\frac{3}{4} = 1.75$).
- 1.7 Write the fraction represented by a drawing of parts of a figure; represent a given fraction by using drawings; and relate a fraction to a simple decimal on a number line.

Which number represents the shaded part of the figure? (Adapted from TIMSS gr. 4, M-5)

1. 2.8
2. 0.5
3. 0.2
4. 0.02



- 1.8** Use concepts of negative numbers (e.g., on a number line, in counting, in temperature, in “owing”).

True or false?

1. $-9 > -10$
2. $-31 < -29$

- 1.9** Identify on a number line the relative position of positive fractions, positive mixed numbers, and positive decimals to two decimal places.

2.0 Students extend their use and understanding of whole numbers to the addition and subtraction of simple decimals:

- 2.1 Estimate and compute the sum or difference of whole numbers and positive decimals to two places.

Solve $55.73 - 48.25 = ?$

- 2.2 Round two-place decimals to one decimal or the nearest whole number and judge the reasonableness of the rounded answer.

Solve $17.91 + 2.18 = ?$

3.0 Students solve problems involving addition, subtraction, multiplication, and division of whole numbers and understand the relationships among the operations:

- 3.1** Demonstrate an understanding of, and the ability to use, standard algorithms for the addition and subtraction of multidigit numbers.

Solve $619,581 - 23,183 = ?$

Solve $6,747 + 321,105 = ?$

Number Sense (Continued)

- 3.2** Demonstrate an understanding of, and the ability to use, standard algorithms for multiplying a multidigit number by a two-digit number and for dividing a multidigit number by a one-digit number; use relationships between them to simplify computations and to check results.

Solve:

1. $783 \times 23 = ?$
2. $8,988/6 = ?$
3. $11,115/9 = ?$

- 3.3** Solve problems involving multiplication of multidigit numbers by two-digit numbers.

- 3.4** Solve problems involving division of multidigit numbers by one-digit numbers.

4.0 Students know how to factor small whole numbers:

- 4.1 Understand that many whole numbers break down in different ways (e.g., $12 = 4 \times 3 = 2 \times 6 = 2 \times 2 \times 3$).

In how many distinct ways can you write 60 as a product of two numbers?

- 4.2** Know that numbers such as 2, 3, 5, 7, and 11 do not have any factors except 1 and themselves and that such numbers are called prime numbers.

List all the distinct prime factors of 210.

Algebra and Functions**1.0 Students use and interpret variables, mathematical symbols, and properties to write and simplify expressions and sentences:**

- 1.1 Use letters, boxes, or other symbols to stand for any number in simple expressions or equations (e.g., demonstrate an understanding and the use of the concept of a variable).

Tanya has read the first 78 pages of a 130-page book. Give the number of the sentence that can be used to find the number of pages Tanya must read to finish the book. (Adapted from TIMSS gr. 4, I-7)

1. $130 + 78 = \underline{\quad}$
2. $\underline{\quad} - 78 = 130$
3. $130 - 78 = \underline{\quad}$
4. $130 - \underline{\quad} = 178$

Algebra and Functions (Continued)

- 1.2** Interpret and evaluate mathematical expressions that now use parentheses.

Evaluate the two expressions: $(28 - 10) - 8 = \underline{\quad}$ and $28 - (10 - 8) = \underline{\quad}$.

- 1.3** Use parentheses to indicate which operation to perform first when writing expressions containing more than two terms and different operations.

Solve $(3 \times 12) - \left(\frac{24}{6} + 8\right) = ?$

Solve $\left(\frac{18 + 31}{7} + 5\right) \times 9 = ?$


- 1.4 Use and interpret formulas (e.g., area = length \times width or $A = lw$) to answer questions about quantities and their relationships.

There are many rules to get from Column A to Column B in the following table. Can you state one rule? (Adapted from TIMSS, gr. 4, J-5)

Column A	Column B
10	2
15	3
45	9
50	10

- 1.5** Understand that an equation such as $y = 3x + 5$ is a prescription for determining a second number when a first number is given.
- 2.0** Students know how to manipulate equations:
- 2.1** Know and understand that equals added to equals are equal.
- 2.2** Know and understand that equals multiplied by equals are equal.

Note: The sample problems illustrate the standards and are written to help clarify them. Some problems are written in a form that can be used directly with students; others will need to be modified, particularly in the primary grades, before they are used with students.


The symbol  identifies the key standards for grade five.

By the end of grade five, students increase their facility with the four basic arithmetic operations applied to fractions and decimals and learn to add and subtract positive and negative numbers. They know and use common measuring units to determine length and area and know and use formulas to determine the volume of simple geometric figures. Students know the concept of angle measurement and use a protractor and compass to solve problems. They use grids, tables, graphs, and charts to record and analyze data.

Number Sense

1.0 Students compute with very large and very small numbers, positive integers, decimals, and fractions and understand the relationship between decimals, fractions, and percents. They understand the relative magnitudes of numbers:

1.1 Estimate, round, and manipulate very large (e.g., millions) and very small (e.g., thousandths) numbers.

 Interpret percents as a part of a hundred; find decimal and percent equivalents for common fractions and explain why they represent the same value; compute a given percent of a whole number.

A test had 48 problems. Joe got 42 correct.

1. What percent were correct?
2. What percent were wrong?
3. If Moe got 93.75% correct, how many problems did he get correct?

1.3 Understand and compute positive integer powers of nonnegative integers; compute examples as repeated multiplication.

Extend the tables shown below:


$$2^4 = 16 \qquad 10^4 = 10,000$$

$$2^3 = 8 \qquad 10^3 = 1,000$$

$$2^2 = 4 \qquad 10^2 = 100$$

$$2^1 = ? \qquad 10^1 = ?$$

$$2^0 = ? \qquad 10^0 = ?$$

 Determine the prime factors of all numbers through 50 and write the numbers as the product of their prime factors by using exponents to show multiples of a factor (e.g., $24 = 2 \times 2 \times 2 \times 3 = 2^3 \times 3$).

Find the prime factorization of 48 and use exponents where appropriate.

Number Sense (Continued)

- 1.5** Identify and represent on a number line decimals, fractions, mixed numbers, and positive and negative integers.

2.0 Students perform calculations and solve problems involving addition, subtraction, and simple multiplication and division of fractions and decimals:

Grade Five

- 2.1** Add, subtract, multiply, and divide with decimals; add with negative integers; subtract positive integers from negative integers; and verify the reasonableness of the results.

Determine the following numbers:

1. $11 + (-23)$
2. $(-15) - 128$
3. $(-27) + (-45)$

- 2.2** Demonstrate proficiency with division, including division with positive decimals and long division with multidigit divisors.

Find the quotient:

6 divided by .025

- 2.3** Solve simple problems, including ones arising in concrete situations, involving the addition and subtraction of fractions and mixed numbers (like and unlike denominators of 20 or less), and express answers in the simplest form.

Given the following three pairs of fractions ($\frac{3}{8}$ and $\frac{1}{6}$, $5\frac{3}{4}$ and $2\frac{1}{3}$, 16 and $12\frac{7}{8}$), find for each pair its:

1. Sum
2. Difference

- 2.4** Understand the concept of multiplication and division of fractions.

- 2.5** Compute and perform simple multiplication and division of fractions and apply these procedures to solving problems.

Given the following three pairs of fractions ($\frac{3}{8}$ and $\frac{1}{6}$, $5\frac{3}{4}$ and $2\frac{1}{3}$, 16 and $12\frac{7}{8}$), find for each pair its:

1. Product
2. Quotient in simplest terms

Ericka has $3\frac{1}{2}$ yards of cloth to make shirts. Each shirt requires $\frac{7}{8}$ yard. How many shirts can she make? How much cloth will she have left over?

Algebra and Functions

1.0 Students use variables in simple expressions, compute the value of the expression for specific values of the variable, and plot and interpret the results:

1.1 Use information taken from a graph or equation to answer questions about a problem situation.

1.2 Use a letter to represent an unknown number; write and evaluate simple algebraic expressions in one variable by substitution.

$$3x + 2 = 14. \text{ What is } x?$$

1.3 Know and use the distributive property in equations and expressions with variables.

1.4 Identify and graph ordered pairs in the four quadrants of the coordinate plane.

Plot the points $(1, 2)$, $(-4, -3)$, $(12, -1)$, $(0, 4)$, $(-4, 0)$.

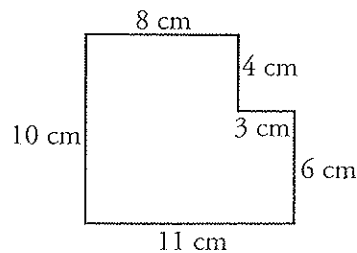
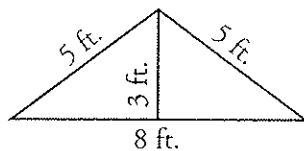
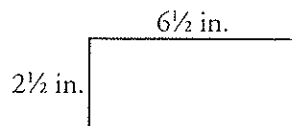
1.5 Solve problems involving linear functions with integer values; write the equation; and graph the resulting ordered pairs of integers on a grid.

Measurement and Geometry

1.0 Students understand and compute the volumes and areas of simple objects:

1.1 Derive and use the formula for the area of a triangle and of a parallelogram by comparing it with the formula for the area of a rectangle (i.e., two of the same triangles make a parallelogram with twice the area; a parallelogram is compared with a rectangle of the same area by pasting and cutting a right triangle on the parallelogram).


Find the area and perimeter.



Grade Six Mathematics Content Standards

By the end of grade six, students have mastered the four arithmetic operations with whole numbers, positive fractions, positive decimals, and positive and negative integers; they accurately compute and solve problems. They apply their knowledge to statistics and probability. Students understand the concepts of mean, median, and mode of data sets and how to calculate the range. They analyze data and sampling processes for possible bias and misleading conclusions; they use addition and multiplication of fractions routinely to calculate the probabilities for compound events. Students conceptually understand and work with ratios and proportions; they compute percentages (e.g., tax, tips, interest). Students know about π and the formulas for the circumference and area of a circle. They use letters for numbers in formulas involving geometric shapes and in ratios to represent an unknown part of an expression. They solve one-step linear equations.

Note: The sample problems illustrate the standards and are written to help clarify them. Some problems are written in a form that can be used directly with students; others will need to be modified, particularly in the primary grades, before they are used with students.

The symbol  identifies the key standards for grade six.

Number Sense

- 1.0** Students compare and order positive and negative fractions, decimals, and mixed numbers. Students solve problems involving fractions, ratios, proportions, and percentages:
- 1.1** Compare and order positive and negative fractions, decimals, and mixed numbers and place them on a number line.
 - 1.2** Interpret and use ratios in different contexts (e.g., batting averages, miles per hour) to show the relative sizes of two quantities, using appropriate notations (a/b , a to b , $a:b$).
- Write the following as ratios:
1. The ratio of tricycles to tricycle wheels
 2. The ratio of hands to fingers
 3. If there are 6 tricycle wheels, how many tricycles are there?
 4. If there are 45 fingers, how many hands are there?
- 1.3** Use proportions to solve problems (e.g., determine the value of N if $4/7 = N/21$, find the length of a side of a polygon similar to a known polygon). Use cross-multiplication as a method for solving such problems, understanding it as the multiplication of both sides of an equation by a multiplicative inverse.

Number Sense (Continued)

Find n if:

1. $\frac{49}{21} = \frac{14}{n}$

2. $\frac{n}{3} = \frac{5}{7}$

(This problem also applies to Algebra and Functions Standard 1.1.)

- 1.4** Calculate given percentages of quantities and solve problems involving discounts at sales, interest earned, and tips.

2.0 Students calculate and solve problems involving addition, subtraction, multiplication, and division:

- 2.1 Solve problems involving addition, subtraction, multiplication, and division of positive fractions and explain why a particular operation was used for a given situation.
- 2.2 Explain the meaning of multiplication and division of positive fractions and perform the calculations (e.g., $\frac{5}{8} \div \frac{15}{16} = \frac{5}{8} \times \frac{16}{15} = \frac{2}{3}$).

1. If $\frac{1}{2}$ is divided by a certain fraction $\frac{1}{6}$, the result is $\frac{3}{8}$. What is $\frac{1}{6}$?2. Draw a rectangle that has a perimeter of 1 and an area of less than $\frac{1}{30}$.

- 2.3** Solve addition, subtraction, multiplication, and division problems, including those arising in concrete situations, that use positive and negative integers and combinations of these operations.

Simplify to make the calculation as simple as possible and identify the properties you used at each step:

1. $95 + 276 + 5$

2. $-19 + 37 + 19$

3. $-16(-8 + 9)$

4. $\left(-\frac{7}{8}\right)\left(\frac{17}{17}\right)$

5. $(-8)(-4)(19)(6 + (-6))$

- 2.4** Determine the least common multiple and the greatest common divisor of whole numbers; use them to solve problems with fractions (e.g., to find a common denominator to add two fractions or to find the reduced form for a fraction).

Algebra and Functions

1.0 Students write verbal expressions and sentences as algebraic expressions and equations; they evaluate algebraic expressions, solve simple linear equations, and graph and interpret their results:

- 1.1** Write and solve one-step linear equations in one variable.

$6y - 2 = 10$. What is y ?

- 1.2 Write and evaluate an algebraic expression for a given situation, using up to three variables.

Joe's sister Mary is twice as old as he is. Mary is 16. How old is Joe?

- 1.3 Apply algebraic order of operations and the commutative, associative, and distributive properties to evaluate expressions; and justify each step in the process.

- 1.4 Solve problems manually by using the correct order of operations or by using a scientific calculator.

2.0 Students analyze and use tables, graphs, and rules to solve problems involving rates and proportions:

- 2.1 Convert one unit of measurement to another (e.g., from feet to miles, from centimeters to inches).

Suppose that one British pound is worth \$1.50. In London a magazine costs 3 pounds. In San Francisco the same magazine costs \$4.25. In which city is the magazine cheaper?

When temperature is measured in both Celsius (C) and Fahrenheit (F), it is known that they are related by the following formula:

$$9 \times C = (F - 32) \times 5.$$

What is 50 degrees Fahrenheit in Celsius? (Note the explicit use of parentheses.)

- 2.2** Demonstrate an understanding that *rate* is a measure of one quantity per unit value of another quantity.

Joe can type 9 words in 8 seconds. At this rate, how many words can he type in 2 minutes?

- 2.3 Solve problems involving rates, average speed, distance, and time.

Marcus took a train from San Francisco to San Jose, a distance of 54 miles. The train took 45 minutes for the trip. What was the average speed of the train?


Algebra and Functions (Continued)**3.0 Students investigate geometric patterns and describe them algebraically:**

- 3.1 Use variables in expressions describing geometric quantities (e.g., $P = 2w + 2l$, $A = \frac{1}{2}bh$, $C = \pi d$ —the formulas for the perimeter of a rectangle, the area of a triangle, and the circumference of a circle, respectively).

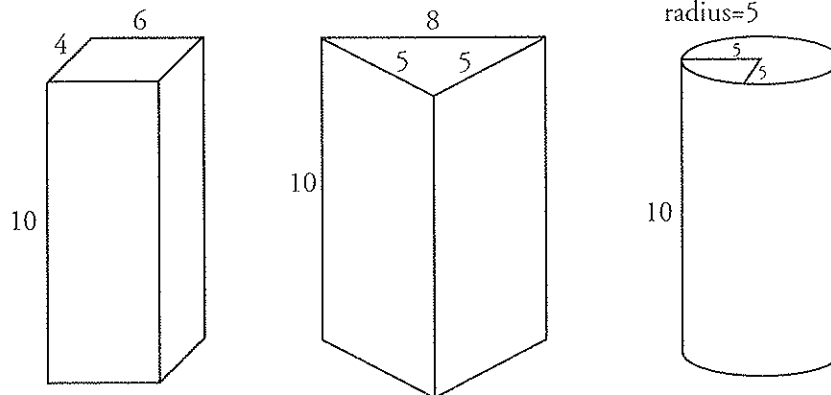
A rectangle has width w . Its length is one more than 3 times its width. Find the perimeter of the rectangle. (Your answer will be expressed in terms of w .)

- 3.2 Express in symbolic form simple relationships arising from geometry.

Measurement and Geometry**1.0 Students deepen their understanding of the measurement of plane and solid shapes and use this understanding to solve problems:**


-  Understand the concept of a constant such as π ; know the formulas for the circumference and area of a circle.
- 1.2 Know common estimates of π (3.14; $\frac{22}{7}$) and use these values to estimate and calculate the circumference and the area of circles; compare with actual measurements.
- 1.3 Know and use the formulas for the volume of triangular prisms and cylinders (area of base \times height); compare these formulas and explain the similarity between them and the formula for the volume of a rectangular solid.

Find the volumes (dimensions are in cm).



Grade Seven Mathematics Content Standards

Note: The sample problems illustrate the standards and are written to help clarify them. Some problems are written in a form that can be used directly with students; others will need to be modified, particularly in the primary grades, before they are used with students.

The symbol  identifies the key standards for grade seven.


By the end of grade seven, students are adept at manipulating numbers and equations and understand the general principles at work. Students understand and use factoring of numerators and denominators and properties of exponents. They know the Pythagorean theorem and solve problems in which they compute the length of an unknown side. Students know how to compute the surface area and volume of basic three-dimensional objects and understand how area and volume change with a change in scale. Students make conversions between different units of measurement. They know and use different representations of fractional numbers (fractions, decimals, and percents) and are proficient at changing from one to another. They increase their facility with ratio and proportion, compute percents of increase and decrease, and compute simple and compound interest. They graph linear functions and understand the idea of slope and its relation to ratio.

Number Sense**1.0 Students know the properties of, and compute with, rational numbers expressed in a variety of forms:**

- 1.1 Read, write, and compare rational numbers in scientific notation (positive and negative powers of 10) with approximate numbers using scientific notation.

Write the following as a power of 10 or the product of a whole number and a power of 10:

1. 10,000
2. Ten billion
3. 6,000,000
4. 3 hundred thousand

-  Add, subtract, multiply, and divide rational numbers (integers, fractions, and terminating decimals) and take positive rational numbers to whole-number powers.

Write the prime factorization of the following numbers:

840 396 605 1,859

- 1.3 Convert fractions to decimals and percents and use these representations in estimations, computations, and applications.

Change to decimals:

$$\frac{7}{8} \quad \frac{7}{11}$$

Number Sense (Continued)

- 1.4** Differentiate between rational and irrational numbers.
- 1.5** Know that every rational number is either a terminating or repeating decimal and be able to convert terminating decimals into reduced fractions.

Change to fractions:

$$0.27 \quad 0.272727$$

Find the period of the repeating part of $\frac{41}{13}$.

- 1.6 Calculate the percentage of increases and decreases of a quantity.
- 1.7** Solve problems that involve discounts, markups, commissions, and profit and compute simple and compound interest.

Joe borrows \$800 at 10% interest compounded every six months. How much interest will there be in 4 years?

2.0 Students use exponents, powers, and roots and use exponents in working with fractions:

- 2.1 Understand negative whole-number exponents. Multiply and divide expressions involving exponents with a common base.

Continue the sequence:

$$2^4 = 16$$

$$2^3 = 8$$

$$2^2 = 4$$

$$2^1 = 2$$

$$2^0 = 1$$

$$2^{-1} = ?$$

$$2^{-2} = ?$$

- 2.2** Add and subtract fractions by using factoring to find common denominators.

Make use of prime factors to compute:

$$1. \frac{2}{28} + \frac{1}{49}$$

$$2. \frac{-5}{63} + \left(\frac{-7}{99}\right)$$

$$3. \left(\frac{42}{22}\right)\left(\frac{75}{63}\right)$$

Number Sense (Continued)

- 2.3** Multiply, divide, and simplify rational numbers by using exponent rules.
- 2.4 Use the inverse relationship between raising to a power and extracting the root of a perfect square integer; for an integer that is not square, determine without a calculator the two integers between which its square root lies and explain why.
- Find the edge of a square which has an area of 81.
- 2.5** Understand the meaning of the absolute value of a number; interpret the absolute value as the distance of the number from zero on a number line; and determine the absolute value of real numbers.

Algebra and Functions

- 1.0 Students express quantitative relationships by using algebraic terminology, expressions, equations, inequalities, and graphs:**
- 1.1 Use variables and appropriate operations to write an expression, an equation, an inequality, or a system of equations or inequalities that represents a verbal description (e.g., three less than a number, half as large as area A).
- Write the following verbal statements as algebraic expressions:
- The square of a is increased by the sum of twice a and 3.
 - The product of $\frac{1}{2}$ of a and 3 is decreased by the quotient of a divided by (-4) .
- 1.2 Use the correct order of operations to evaluate algebraic expressions such as $3(2x + 5)^2$.
- Given $x = (-2)$ and $y = 5$ evaluate:
- $x^2 + 2x - 3$
 - $\frac{y(xy - 7)}{10}$

Algebra and Functions (Continued)

- 1.3** Simplify numerical expressions by applying properties of rational numbers (e.g., identity, inverse, distributive, associative, commutative) and justify the process used.

Name the property illustrated by each of the following:

1. $x(y + -y) = x(0)$
2. $x(y + -y) = xy + x(-y)$
3. $x(y + -y) = (y + -y)(x)$
4. $x(y + -y) = x(-y + y)$
5. $x(y(1/y)) = x(1)$

- 1.4 Use algebraic terminology (e.g., variable, equation, term, coefficient, inequality, expression, constant) correctly.
- 1.5 Represent quantitative relationships graphically and interpret the meaning of a specific part of a graph in the situation represented by the graph.

2.0 Students interpret and evaluate expressions involving integer powers and simple roots:

- 2.1 Interpret positive whole-number powers as repeated multiplication and negative whole-number powers as repeated division or multiplication by the multiplicative inverse. Simplify and evaluate expressions that include exponents.
- 2.2 Multiply and divide monomials; extend the process of taking powers and extracting roots to monomials when the latter results in a monomial with an integer exponent.

3.0 Students graph and interpret linear and some nonlinear functions:

- 3.1 Graph functions of the form $y = nx^2$ and $y = nx^3$ and use in solving problems.
- 3.2 Plot the values from the volumes of three-dimensional shapes for various values of the edge lengths (e.g., cubes with varying edge lengths or a triangle prism with a fixed height and an equilateral triangle base of varying lengths).

- 3.3** Graph linear functions, noting that the vertical change (change in y -value) per unit of horizontal change (change in x -value) is always the same and know that the ratio ("rise over run") is called the slope of a graph.

- 3.4** Plot the values of quantities whose ratios are always the same (e.g., cost to the number of an item, feet to inches, circumference to diameter of a circle). Fit a line to the plot and understand that the slope of the line equals the quantities.

Algebra and Functions (Continued)

- 4.0** Students solve simple linear equations and inequalities over the rational numbers:
- 4.1** Solve two-step linear equations and inequalities in one variable over the rational numbers, interpret the solution or solutions in the context from which they arose, and verify the reasonableness of the results.
- 4.2** Solve multistep problems involving rate, average speed, distance, and time or a direct variation.

Measurement and Geometry

- 1.0** Students choose appropriate units of measure and use ratios to convert within and between measurement systems to solve problems:
- 1.1 Compare weights, capacities, geometric measures, times, and temperatures within and between measurement systems (e.g., miles per hour and feet per second, cubic inches to cubic centimeters).
- Convert the following:
- 80 miles/hr. = ? ft./sec.
 - 20 oz./min. = ? qts./day
- 1.2 Construct and read drawings and models made to scale.
- 1.3** Use measures expressed as rates (e.g., speed, density) and measures expressed as products (e.g., person-days) to solve problems; check the units of the solutions; and use dimensional analysis to check the reasonableness of the answer.
- 2.0** Students compute the perimeter, area, and volume of common geometric objects and use the results to find measures of less common objects. They know how perimeter, area, and volume are affected by changes of scale:
- 2.1 Use formulas routinely for finding the perimeter and area of basic two-dimensional figures and the surface area and volume of basic three-dimensional figures, including rectangles, parallelograms, trapezoids, squares, triangles, circles, prisms, and cylinders.
- 2.2 Estimate and compute the area of more complex or irregular two- and three-dimensional figures by breaking the figures down into more basic geometric objects.