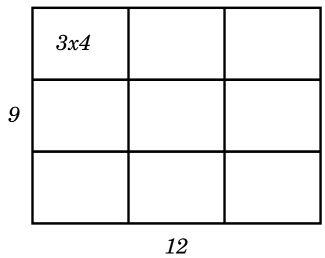
MATH 100 **February 19, 2008**

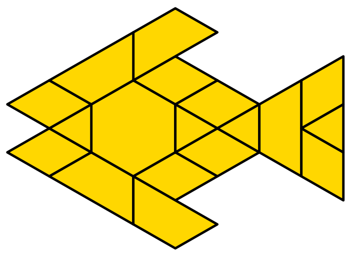
Test 1 - Solutions

1. A first-grade teacher is going to have her students make a number of paper dinosaurs for a counting project. She needs one hundred 3 in $\times$ 4 in rectangles, on which students will trace and then cut out dinosaurs. If her construction paper has dimensions 9 in $\times$ 12 in, how many sheets of paper will she need for this project?



As shown in the picture above, the teacher can cut a 9 in $\times$ 12 in sheet of paper into nine 3 in $\times$ 4 in rectangles. Since $\frac{100}{9} = 11\frac{1}{9}$*, she needs 12 sheets of paper.*

2. If the area of the hexagon is 1 square unit, what is the area of the fish shown below?



The fish figure consists of 1 hexagon, 5 trapezoids, 8 parallelograms, and 4 triangles. The area of each trapezoid is $\frac{1}{2}$ *square units, the area of each parallelogram is* $\frac{1}{3}$ *square units, and the area of each tringle is* $\frac{1}{6}$ *square units. Thus the total area is* $1 + \frac{5}{2} + \frac{8}{3} + \frac{4}{6} = \frac{6}{6} + \frac{15}{6} + \frac{16}{6} + \frac{4}{6} = \frac{6+15+16+4}{6} = \frac{41}{6} = 6\frac{5}{6}$ *square units.*

For extra credit: what is the perimeter of this fish?

Let a be the length of each side of the triangles, parallelograms, and hexagons. (Then three sides of a trapezoid have length a and one side has length $2a$.) The area of an equilateral triangle with side 1 unit is $\frac{\sqrt{3}}{4}$ *square units. Then the area of an equilateral triangle with side a units is* $\frac{\sqrt{3}a^2}{4}$ *units. Since the area of each triangle in the picture is* $\frac{1}{6}$ *square units, we have* $\frac{\sqrt{3}a^2}{4} = \frac{1}{6}$*. Multiplying both sides by* $\frac{4}{\sqrt{3}}$ *gives* $a^2 = \frac{4}{6\sqrt{3}} = \frac{2}{3\sqrt{3}}$*. Then* $a = \frac{\sqrt{2}}{\sqrt{3\sqrt{3}}}$*. The perimeter of the fish is* $25a = \frac{25\sqrt{2}}{\sqrt{3\sqrt{3}}}$.

3. Let the side of each of the small squares be 1 unit. Find the number of squares in, the area, and the perimeter of each of the shown figures. Notice the pattern and use it to predict the above quantities for the 50th figure in this sequence.

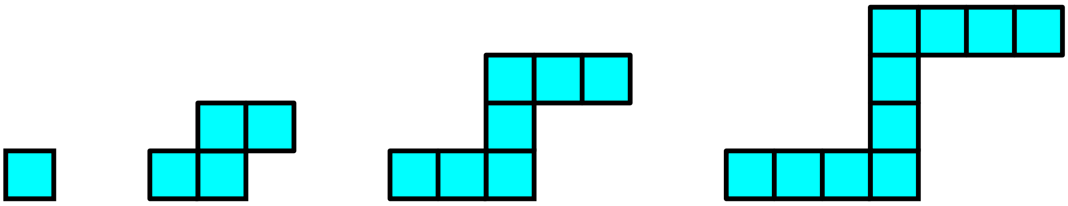


Figure in sequence	Number of squares	Area	Perimeter
1st	*1*	*1*	*4*
2nd	*4*	*4*	*10*
3rd	*7*	*7*	*16*
4th	*10*	*10*	*22*
50th	*148*	*148*	*298*

Pattern: the number of squares in the figure always increases by 3. So does the area (the area is equal to the number of squares since the area of each square is 1 square unit). The perimeter increases by 6.

For extra credit: find formulas for the above quantities for the n-th figure in the sequence.

Using the pattern described above, we obtain that the n-th figure consists of $1+3(n-1) = 3n-2$ *squares. Its area is* $3n-2$ *square units. Its perimeter is* $4+6(n-1) = 6n-2$ *units.*

4. Bob and Cindy bought a pizza. Bob ate $\frac{1}{3}$ of the pizza and Cindy ate $\frac{1}{4}$ of the pizza.

   (a) How much pizza was left?

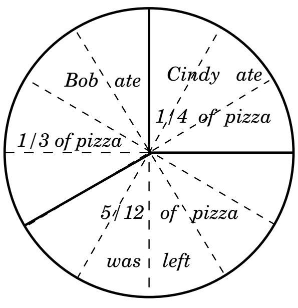
   They ate $\frac{1}{3}+\frac{1}{4} = \frac{4}{12}+\frac{3}{12} = \frac{7}{12}$ *of the pizza, so* $1-\frac{7}{12} = \frac{12}{12}-\frac{7}{12} = \frac{5}{12}$ *of the pizza was left.*

   (b) If the weight of the remaining pizza is $\frac{1}{2}$ lb, what was the weight of the whole pizza?

   The weight of the whole pizza was $\frac{1}{2} \div \frac{5}{12} = \frac{1}{2} \times \frac{12}{5} = \frac{12}{10} = \frac{6}{5}$ *lb.*

   For extra credit: show how you can check your answers.

   To check our answer to part (a), we can use a picture:



To check our answer to part (b), we will calculate the weight of $\frac{1}{12}$ *of pizza (one slice on the picture) first. If the weigth of* $\frac{5}{12}$ *of pizza (5 slices) is* $\frac{1}{2} = \frac{5}{10}$ *lb, then the weight of* $\frac{1}{12}$ *of pizza (one slice) is* $\frac{1}{10}$ *lb. Then the weigth of the whole pizza (12 slices) is* $\frac{12}{10} = \frac{6}{5}$ *lb.*