

MATH 110

Homework 10

Explanations are not requested for this homework, stating your answers clearly will be sufficient.

1. (20%) Determine which of the following pairs (X, τ) are topological spaces. For those that are not, identify all axioms (out of the four axioms in the definition of a topological space) that do not hold.
 - (a) $X = \{1, 2, 3\}$, $\tau = \{\emptyset\}$
 - (b) $X = \{1, 2, 3\}$, $\tau = \{\{1\}, \{1, 2\}, \{1, 2, 3\}\}$
 - (c) $X = \{1, 2, 3\}$, $\tau = \{\emptyset, \{1\}, \{2\}, \{1, 2, 3\}\}$
 - (d) $X = \{1, 2, 3\}$, $\tau = \{\emptyset, \{2\}, \{1, 2, 3\}\}$
 - (e) $X = \{1, 2, 3\}$, $\tau = \{\emptyset, \{1, 2\}, \{1, 3\}, \{1, 2, 3\}\}$
2. (80%) Consider the set \mathbb{R} with the usual topology. For each subset of \mathbb{R} given below,
 - determine whether it is open, closed, both, or neither; and
 - find its interior and closure.
 - (a) \emptyset
 - (b) \mathbb{R}
 - (c) $\{0\}$
 - (d) $\mathbb{R} - \{0\}$
 - (e) \mathbb{Z} (the set of all integer numbers)
 - (f) \mathbb{Q} (the set of all rational numbers)
 - (g) $(2, 3) \cup (4, \infty)$
 - (h) $[2, 3) \cup (3, 4]$
 - (i) $(2, 3) \cup \{4\}$
 - (j) $[2, 3] \cup \{4\}$