MATH 110

Fundamental Identities in Set Theory

Let U be the universal set and A, B, and C be any subsets of U. The following identities can be verified (illustrated) using Venn diagrams, and formally proved by showing that each side is a subset of the other.

1. Commutative laws:

$$A \cup B = B \cup A$$
$$A \cap B = B \cap A$$

2. Associative laws:

$$(A \cup B) \cup C = A \cup (B \cup C)$$
$$(A \cap B) \cap C = A \cap (B \cap C)$$

3. Distributive laws:

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$
$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

4. Idempotent laws:

1	$A \cup A = A$
	$A \cap A = A$
5. Identity laws:	
	$A \cup \emptyset = A$
	$A \cap U = A$
6. Inverse laws:	
	$A\cup \overline{A}=U$
	$A\cap \overline{A}=\emptyset$
7. Domination laws:	
	$A\cup U=U$
	$A \cap \emptyset = \emptyset$
8. Absorption laws:	
-	$A \cup (A \cap B) = A$
	$A \cap (A \cup B) = A$

9. Double complement law:

$$\overline{(\overline{A})} = A$$

10. DeMorgan's laws:

$$\overline{A \cup B} = \overline{A} \cap \overline{B}$$
$$\overline{A \cap B} = \overline{A} \cup \overline{B}$$

11. Difference identity:

$$A - B = A \cap \overline{B}$$

12. "Contrapositive" identity:

$$A - B = \overline{B} - \overline{A}$$

13. Symmetric difference identities:

$$(A - B) \cup (B - A) = (A \cup B) \cap (\overline{A} \cup \overline{B})$$
$$(A - B) \cup (B - A) = (A \cup B) - (A \cap B)$$