Similarities between logical and set operations

Observe that if in a logical equivalence containing only operations negation, conjunction, and disjunction, each negation is replaced by complement, each conjunction is replaced by intersection, each conjunction is replaced by union, each F is replaced by the emtpy set, and each T is replaced by the universal set, then a set identity is obtained. Identities involving implication and biconditional are a bit trickier and will be discussed below (in the last three items of the list below).

In the following identities, P, Q, and R are propositional variables, set U is the universal set, and A, B, and C are any subsets of U.

1. Commutative laws:

$$P \vee Q \equiv Q \vee P \qquad A \cup B = B \cup A$$

$$P \wedge Q \equiv Q \wedge P \qquad A \cap B = B \cap A$$

2. Associative laws:

$$(P \vee Q) \vee R \equiv P \vee (Q \vee R) \qquad (A \cup B) \cup C = A \cup (B \cup C)$$

$$(P \wedge Q) \wedge R \equiv P \wedge (Q \wedge R) \qquad (A \cap B) \cap C = A \cap (B \cap C)$$

3. Distributive laws:

$$P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R) \qquad A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

$$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R) \qquad A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

4. Idempotent laws:

$$P \vee P \equiv P \qquad A \cup A = A$$

$$P \wedge P \equiv P \qquad A \cap A = A$$

5. Identity laws:

$$P \vee F \equiv P \qquad A \cup \emptyset = A$$

$$P \wedge T \equiv P \qquad A \cap U = A$$

6. Inverse laws:

$$P \vee \neg P \equiv T \qquad A \cup \overline{A} = U$$

$$P \wedge \neg P \equiv F \qquad A \cap \overline{A} = \emptyset$$

7. Domination laws:

$$P \vee T \equiv T \qquad\qquad A \cup U = U$$

$$P \wedge F \equiv F \qquad\qquad A \cap \emptyset = \emptyset$$

8. Absorption laws:

$$P \vee (P \wedge Q) \equiv P \qquad\qquad A \cup (A \cap B) = A$$

$$P \wedge (P \vee Q) \equiv P \qquad\qquad A \cap (A \cup B) = A$$

9. Double negation/double complement law:

$$\neg(\neg P) \equiv P \qquad\qquad \overline{(\overline{A})} = A$$

10. DeMorgan's laws:

$$\neg(P \vee Q) \equiv (\neg P) \wedge (\neg Q) \qquad\qquad \overline{A \cup B} = \overline{A} \cap \overline{B}$$

$$\neg(P \wedge Q) \equiv (\neg P) \vee (\neg Q) \qquad\qquad \overline{A \cap B} = \overline{A} \cup \overline{B}$$

11. Implication identity:

$$P \to Q \equiv (\neg P) \vee Q$$

Since $P \to Q$ is false only when P is true and Q is false, and is true otherwise, considering the corresponding Venn diagram, we see that it corresponds to $\overline{A - B}$. Thus the above implication identity gives

$$\overline{A - B} = \overline{A} \cup B.$$

This is equivalent to the following. Difference identity:

$$A - B = A \cap \overline{B}$$

12. Contrapositive identity:

$$P \to Q \equiv (\neg Q) \to (\neg P)$$

This gives

$$\overline{A - B} = \overline{\overline{B} - \overline{A}}$$

which is equivalent to

$$A - B = \overline{B} - \overline{A}$$

13. Biconditional identities:

$$P \leftrightarrow Q \equiv (P \wedge Q) \vee ((\neg P) \wedge (\neg Q))$$

$$P \leftrightarrow Q \equiv (P \rightarrow Q) \wedge (Q \rightarrow P)$$

The first identity gives

$$\overline{(A - B) \cup (B - A)} = (A \cap B) \cup (\overline{A} \cap \overline{B})$$

which is equivalent to

$$\begin{aligned}(A - B) \cup (B - A) &= \overline{A \cap B} \cap \overline{\overline{A} \cap \overline{B}} \\ &= (\overline{A} \cup \overline{B}) \cap (A \cup B)\end{aligned}$$

which gives the first of the symmetric difference identities below. Also,

$$\begin{aligned}P \leftrightarrow Q &\equiv (P \wedge Q) \vee ((\neg P) \wedge (\neg Q)) \\ &\equiv \neg(P \vee Q) \vee (P \wedge Q) \\ &\equiv (P \vee Q) \rightarrow (P \wedge Q)\end{aligned}$$

gives

$$\overline{(A - B) \cup (B - A)} = \overline{(A \cup B) - (A \cap B)}$$

which implies the second of the symmetric difference identities below.
Symmetric difference identities:

$$(A - B) \cup (B - A) = (A \cup B) \cap (\overline{A} \cup \overline{B})$$

$$(A - B) \cup (B - A) = (A \cup B) - (A \cap B)$$