Math 111 **Name:** ____________________________

May 15, 2006 **Final Exam**

Attempt all 7 problems. Unsupported work will receive no credit, and partially completed work will receive partial credit. Please turn off your cell phone! Good luck!

1. (15 points) Prove that for every odd integer n, $6n^2 + 5n + 4$ is odd.

2. Make truth tables for the following compound statements.

   (a) (10 points) $Q \vee (R \wedge S)$

   (b) (10 points) $(P \wedge (P \iff Q)) \wedge \sim Q$

3. Provide counterexamples to the following proposed (but false) statements.

   (a) (5 points) $\forall x \in \mathbb{R}, \forall y \in \mathbb{R}, (x > 1 \wedge y > 0) \implies y^x > x$.

   (b) (5 points) For all positive integers x, $x^2 - x + 11$ is a prime number.

4. (20 points) A sequence $\{x_n\}$ is defined recursively by $x_1 = 1$, $x_2 = 2$, and $x_n = x_{n-1} + 2x_{n-2}$ for $n \geq 3$. Conjecture a formula for x_n and verify that your conjecture is correct.

5. A relation R is defined on $\mathbb{Z}$ by $x \ R \ y$ if $x \cdot y \geq 0$. Prove or disprove the following:

   (a) (8 points) R is reflexive,

   (b) (8 points) R is symmetric,

   (c) (9 points) R is transitive.

6. Let A, B, and C be sets.

   (a) (20 points) Prove that $A \subseteq B$ iff $A - B = \emptyset$.

   (b) (20 points) Prove that if $A \subseteq B \cup C$ and $A \cap B = \emptyset$, then $A \subseteq C$.

7. (20 points) Prove that $f : \mathbb{R} \to \mathbb{R}$ given by

$$f(x) = \begin{cases} x+4 & \text{if } x \leq -2 \\ -x & \text{if } -2 < x < 2 \\ x-4 & \text{if } x \geq 2 \end{cases}$$

is onto $\mathbb{R}$ but not one-to-one. (*Hint:* Try to graph this function; this will help you see how to prove what you need to prove.)

Extra Credit

(15 points) Let $f : \mathbb{N} \times \mathbb{N} \to \mathbb{N}$ be a function given by $f((m, n)) = 2^{m-1}(2n-1)$. Is f one-to-one? Is f onto?