MATH 111 **Fall 2007**

Homework 3 (due Wed/Thur, Sep 19/20)

Try to do this homework by Sep 19/20, but you may turn it in the following week (by Sep 26/27).

Do exercises 2.14, 2.16, and 2.22 in the book, and the following problems:

1. Let $P(x)$ denote "$x = -2$" and let $Q(x)$ denote "$x^2 = 4$" (as in exercise 2.16 in the book). Determine (and explain!) the truth values of the following statements:
   (a) $\exists x \neg P(x)$
   (b) $\forall x (P(x) \vee Q(x))$
   (c) $\exists x (P(x) \wedge Q(x))$
   (d) $\forall x (P(x) \Rightarrow Q(x))$
   (e) $\exists x (Q(x) \Rightarrow P(x))$
   (f) $\forall x (P(x) \Leftrightarrow Q(x))$

2. Let $F(x, y)$ be statement "x can fool y", where the domain for both variables is the set of all people in the world. Use quantifiers to express each of the following statements:
   (a) Mike can fool everybody.
   (b) Everybody can fool somebody.
   (c) No one can fool both Fred and Jerry.

3. Let $Q(x, y)$ denote "$x + y = 0$". What are the truth values of the statements $\exists y \forall x Q(x, y)$ and $\forall x \exists y Q(x, y)$? Explain your answers!

4. Rewrite each of the following statements so that negations appear only within predicates:
   (a) $\neg \forall y \exists x P(x, y)$
   (b) $\neg (\exists x \exists y \neg P(x, y) \wedge \forall x \forall y Q(x, y))$

5. Are propositions
   (a) $\forall x (P(x) \Leftrightarrow Q(x))$ and $(\forall x P(x)) \Leftrightarrow (\forall x Q(x))$
   (b) $\exists x (P(x) \Leftrightarrow Q(x))$ and $(\exists x P(x)) \Leftrightarrow (\exists x Q(x))$

   logically equivalent, i.e. do they always have the same truth value (regardless of what P and Q are)? If so, explain why. If not, give an example of propositional functions $P(x)$ and $Q(x)$ for which one of the propositions is true and the other one is false.