MATH 111 **Fall 2007**

Practice Test 1

Note: the actual test will consist of five or six questions (some with two or three parts).

1. Review all terms, notations, and types of proofs studied in chapters 0–3.
2. Let $U = \{x \in \mathbb{Z} \mid 0 \leq x \leq << 10\}$ be the universal set, $A = \{x \in U \mid x \text{ is even}\}$, $B = \{1, 2, 3, 4, 5\}$.
   (a) Draw a Venn diargram that illustrates the above sets.
   (b) Determine (i.e. list all the elements of) the following sets: $A \cap B$, $\overline{A}$, $A \cup \overline{B}$.
   (c) How many elements does $A \times B$ have?
   (d) List any three elements of $A \times B$.
3. Let $A = \{1\}$, $B = \{2\}$, $C = \{\{3\}\}$, $D = \{1, \{2\}, \{1, 2, 3\}\}$.
   (a) Which of the following statements are true: $A \in D$, $A \subset D$, $B \in D$, $B \subset D$, $C \in D$, $C \subset D$, $\emptyset \in D$, $\emptyset \subset D$?
   (b) What are the cardinalities of these four sets?
4. Let $A_n = \left[\frac{1}{n}, \frac{n+1}{n}\right)$ for each $n \in \mathbb{N}$. Determine $\cup_{n\in\mathbb{N}} A_n$ and $\cap_{n\in\mathbb{N}} A_n$ (no formal proof is required, but please provide an explanation of your answer; a picture might be helpful).
5. (a) Show that $P \Leftrightarrow Q$ and $(P \wedge Q) \vee ((\neg P) \wedge (\neg Q))$ are logically equivalent.
   (b) The compound statement $(P \Leftrightarrow Q) \Leftrightarrow ((P \wedge Q) \vee ((\neg P) \wedge (\neg Q)))$ is a ____________________.
   (c) The compound statement $(P \Leftrightarrow Q) \Leftrightarrow \neg((P \wedge Q) \vee ((\neg P) \wedge (\neg Q)))$ is a ____________________.
6. Determine the truth values of the following statements (where $x,\ y,\ z \in \mathbb{R}$).
   (a) $\exists! x\ (x^2 = 8)$
   (b) $\forall x \exists y\ (xy = 0)$
   (c) $\forall x \exists! y\ (xy = 0)$
   (d) $\exists x \forall y\ (xy = 0)$
   (e) $\exists! x \forall y\ (xy = 0)$
   (f) $\forall x \forall z \exists y\ (x + y = z)$
   (g) $\forall x \exists y \forall z\ (x + y = z)$

7. For each of the following expressions, give an example of a propositional function $P(x, y)$ that makes the statement true; and (a different, of course) example of $P(x, y)$ that makes the statement false. Explain why your examples satisfy the requirements!

   (a) $\exists x \exists y P(x, y)$

   (b) $\exists x \forall y P(x, y)$

   (c) $\forall x \exists y P(x, y)$

   (d) $\forall x \forall y P(x, y)$

8. Let n and m be integers. Prove the following statements and state what types of proof you used.

   (a) If $3n^2 + 5n$ is odd, then $n \geq 10$.

   (b) If n is even, then $3n^2 - 2n - 5$ is odd.

   (c) If $n - 5m$ is odd, then n and m are of the opposite parity.

9. Let x be a real number. Prove the following statements and state what types of proof you used.

   (a) If $x > -7$, then $-5 - x^2 < 0$.

   (b) If $|x| = 5$, then $x^2 + x + 1 > 20$.