MATH 111 **Fall 2007**

Practice Test 2

Note: the actual test will consist of five or six questions.

1. This test is primarily on chapters 4-6, however, knowledge of previously covered material may be required. Review all terms, notations, and types of proofs in chapters 0–6.
2. Prove the following statements. Indicate what type of proof (direct, by contrapositive, or by contradiction) you used.
   (a) If n is an integer such that $5|(n-1)$, then $5|(n^3+n-2)$.
   (b) The number $\log_3 2$ is irrational.
   (c) Let $n \in \mathbb{Z}$. If $7n^2+4$ is even, then n is even.
   (d) Let $x \in \mathbb{R}$. If $2x > x^2 + x^3$, then $x < 1$.
   (e) Let $m, n \in \mathbb{Z}$. Then $3|(mn)$ if and only if $3|m$ or $3|n$.
   (f) The product of a nonzero rational number and an irrational number is irrational.
   (g) Let $a, b, c \in \mathbb{Z}$. If $a \not| (bc)$, then $a \not| b$ and $a \not| c$.
   (h) Let A and B be sets. Then $A \cap B = \emptyset$ if and only if $(A \times B) \cap (B \times A) = \emptyset$.
3. Prove or disprove the following statements.
   (a) There exists a nonzero integer a such that for every real number b, $b^2 \geq a$.
   (b) There exists an integer a such that $a^3 + 2a + 3 = 100$.
   (c) For any integer a there exists an integer b such that $b^2 = a$.
   (d) The sum of any two positive irrational numbers is irrational.
   (e) Any irrational number is the sum of an irrational number and a positive rational number.
   (f) For any sets A and B there exists a set C such that $A \cup C = B \cup C$.
   (g) Let A, B, C, and D be sets such that $A \subset C$ and $B \subset D$. If $A \cap B = \emptyset$, then $C \cap D = \emptyset$.
   (h) Let A, B, C, and D be sets such that $A \subset C$ and $B \subset D$. If $C \cap D = \emptyset$, then $A \cap B = \emptyset$.