

MATH 111

Test 1

October 2, 2007

Name: _____

- No books, notes, or calculators are allowed.
- Please show all your work.

1. (9 points) Let $A = \{x \in \mathbb{N} \mid x \geq 4\}$ and $B = \{x \in \mathbb{N} \mid 3 \leq x \leq 6\}$, and let \mathbb{N} be the universal set. Determine the following sets:

(a) $A \cap B$

(b) \overline{A}

(c) $\overline{A} \cup B$

2. (7 points) Let P and Q be propositions. Prove that the compound propositions $Q \Rightarrow \neg P$ and $\neg(P \wedge Q)$ are logically equivalent.

3. (9 points) Let $C(x, y)$ denote “ x and y are taking a class together” where x and y are students at Fresno State. Write in words the following statements and determine their truth values. Explain your reasons!

(a) $\forall x \forall y C(x, y)$

(b) $\exists x \forall y C(x, y)$

(c) $\forall x \exists y C(x, y)$

4. (4 points) Let $S = \{1, 2\}$ and $T = \{2, 3, 4\}$. List the elements of $S \times T$.

5. (21 points) Prove the following propositions. For each one, indicate what type of proof you are using.

- Let $n \in \mathbb{Z}$. If $4n + 5$ is even, then $7n^2 - 3n + 8$ is odd.

- Let $n \in \mathbb{Z}$. If n is odd, then $3n + 5$ is even.

- Let $n \in \mathbb{Z}$. If $3n + 5$ is even, then n is odd.

6. (For extra credit, 8 points) If $|A| = 21$, $|B| = 19$, $|C| = 17$, $|A \cap B| = 9$, $|A \cap C| = 8$, $|B \cap C| = 7$, $|(A \cap B) - C| = 6$, find $|A \cup B \cup C|$.