MATH 111

Test 1

October 1, 2007

Name:

- No books, notes, or calculators are allowed.

- Please show all your work.

1. (9 points) Let $A = \{x \in \mathbb{Z} \mid 0 \leq x \leq 3\}$ and $B = \{x \in \mathbb{Z} \mid x \leq 2\}$, and let $\mathbb{Z}$ be the universal set. Determine the following sets:

   (a) $A \cap B$

   (b) $\overline{B}$

   (c) $A \cup \overline{B}$

2. (7 points) Let P and Q be propositions. Prove that the compound propositions $\neg(P \wedge Q)$ and $\neg P \vee \neg Q$ are logically equivalent.

3. (12 points) Let $N(x, y)$ denote "x knows y's name" where x and y are students at Fresno State. Write in words the following statements and predict their truth values. Explain your reasons!

(a) $\forall x \forall y N(x, y)$

(b) $\exists x \forall y N(x, y)$

(c) $\forall x \exists y N(x, y)$

(d) $\forall x \forall y (N(x, y) \Rightarrow N(y, x))$

4. (4 points) Let S and T be sets. Draw a Venn diagram of $S - T$.

5. (8 points) Which of the following propositions can be proved using a vacuous proof? Prove it (use a vacuous proof).

- Let $n \in \mathbb{Z}$. If $4n + 5$ is even, then $7n + 8$ is odd.
- Let $n \in \mathbb{Z}$. If $7n + 8$ is odd, then $4n + 5$ is even.
- Let $n \in \mathbb{Z}$. Then $4n + 5$ is even if and only if $7n + 8$ is odd.

6. (10 points) Let $n \in \mathbb{N}$. Prove that $3n + 7$ is odd if and only if n is even.

7. (**For extra credit**, 8 points) Prove that if sets A and B are disjoint, then $|\mathcal{P}(A \cup B)| = |\mathcal{P}(A)| \cdot |\mathcal{P}(B)|$. What is $|\mathcal{P}(A \cap B)|$ in this case?